Bus Rapid Transit Network Design: Planning Routes and Upgrading Stops Among Multiple Municipalities

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Abstract: Bus Rapid Transit (BRT) lines provide a fast and reliable alternative to regular bus systems at a lower cost than building rail infrastructure. They can enhance public transportation and thus support more sustainable travel. However, BRT lines often require upgraded stops, meaning additional investments are necessary. BRT lines typically span multiple municipalities responsible for the investments, often with varying interests. In this paper, we address the problem of selecting optimal routes and stops for BRT lines considering municipal interests. Moreover, we evaluate our proposed model through a case study in Greater Copenhagen, highlighting the relevance of considering the different municipalities.

Keywords: Network Design ○ Optimisation ○ Bus Rapid Transit ○ Public Transport

1 Introduction

Bus Rapid Transit (BRT) lines have been introduced in many areas to improve public transit due to their reliability, high speeds, service frequency and relatively low investment costs (Deng & Nelson (2011)). However, BRT stops typically require longer platforms, multiple loading areas and higher capacities compared to regular bus stops (Kathuria et al. (2016)). This makes infrastructure investments necessary to accommodate new BRT lines.

In many instances, a BRT line traverses multiple municipalities, which share responsibility for the necessary infrastructure investments (Allansson et al. (2023)). However, not all effects of these investments are perceived equally by local actors, such as inhabitants, businesses and municipalities (Bowes & Ihlanfeldt (2001)). Additionally, to decrease travel times, BRT lines often have fewer stops than regular bus lines. This impacts not only the municipality where a stop is skipped but also others due to the demand to and from that stop. Considering the interests of the different actors involved in such infrastructure investment projects can increase the potential for a proposal to make it through the multitude of planning stages.

In Transit Network Design (TND), several studies have addressed competition or coordination among different actors. For instance, Laporte et al. (2010) used a game-theoretic framework to design robust rail transit networks considering competing transportation modes. Van Der Weijde et al. (2013) examined different interests among public transport operators within a competitive framework to determine optimal transit fares. Rosenthal (2017) applied cooperative game theory to allocate costs in a rapid-transit network, ensuring equitable cost-sharing among users.

Although these studies consider the different interests of actors in TND, they do not include the involvement of regional stakeholders such as municipalities, which play a crucial role in planning BRT lines. At a broader level, competitive aspects of Network Design (ND) involving regional actors have received limited attention. Wang & Zhang (2017) proposed a game-theoretic framework to solve a ND-problem related to increasing road capacities in multiple investment regions. Although the stakes of the different regions are included, their model is not directly applicable to TND, which requires selecting routes and stops.

An example of research that incorporates different municipalities to some extent in TND is the study by Hoogervorst et al. (2022), who studied the edge investment problem for BRT lines. They proposed a model for upgrading segments of a single BRT line, considering the trade-off between attracting passengers and the budget constraints of the investing municipalities. The modelling of this problem was further studied in a bi-objective formulation incorporating both ridership maximisation and investment cost minimisation (Hoogervorst et al. (2024)). A continuous and discrete passenger attraction function was considered to represent traveller response to upgrades. Municipal interests were considered through separate municipality budgets. The routes and stops, which substantially influence investment levels and accessibility, were given as model inputs in both studies, rather than being part of the design process.

In this paper, we propose a (0,1)-Integer Linear Problem (ILP) formulation for the ND-problem applied to BRT systems. It contributes to previous work on ND for BRT systems by (i) considering the choice of stop localisation in settings with multiple municipalities; (ii) defining the utility-based measure of satisfaction for each municipality beyond just budget and investment costs; (iii) introducing equity concepts to incorporate municipal competition and cooperation; and (iv) presenting a formulation for developing multiple BRT lines simultaneously. We evaluate our model through a case study on the Greater Copenhagen region in Denmark.

2 Problem Description and Model Formulation

In this section, we propose a BRT investment problem incorporating municipal interests. We do this by considering the stakes of each municipality in our objective function. To this end, we provide a $(0,1)$-ILP formulation. The notation used in our model is presented in Table 1.

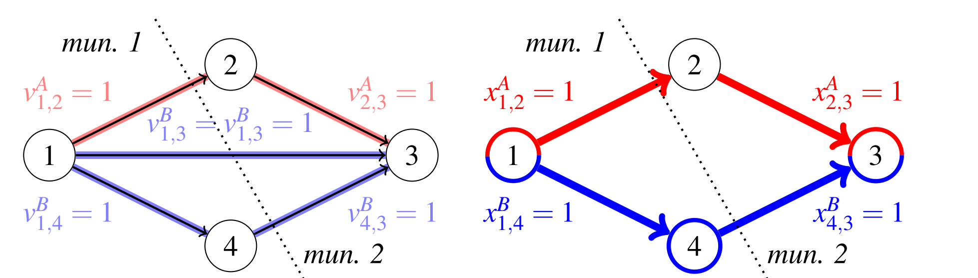
Table 1: Notation

Sets:	
M	Set of municipalities
S	Set of stops
L^{up}	Set of existing lines to upgrade to BRT lines

L	Set of BRT lines to construct from L^{up}
M^l	Set of municipalities that line $l \in L$ crosses
Input:	
k_i^m	Binary parameter equal to 1 if stop $i \in S$ is located in municipality $m \in M$, 0 otherwise
r_i	Binary parameter equal to 1 if an already existing line in L^{up} uses stop $i \in S$, 0 otherwise
$\rho_{o,d}$	Binary parameter equal to 1 if an already existing line in L^{up} directly covers the demand between $o, d \in S$, 0 otherwise
$v_{i,j}^l$	Binary parameter equal to 1 if line $l \in L$ can traverse edge (i, j) for stops $i, j \in S$, 0 otherwise
s^l	The first stop for line $l \in L$
t^l	The final stop for line $l \in L$
$q_{o,d}$	The direct demand between origin stop $o \in S$ and destination stop $d \in S$
c_i^{upgr}	The cost of upgrading stop $i \in S$
b_m	The budget of municipality $m \in M$
$\#N_{\max}^l$	The maximum number of stops that line $l \in L$ may stop at
Variables:	
$x_{i,j}^l$	Binary variable equal to 1 if BRT line $l \in L$ traverses edge (i, j) for $i, j \in S$, 0 otherwise
y_i^l	Binary variable equal to 1 if BRT line $l \in L$ stops at stop $i \in S$, 0 otherwise
γ_i	Binary variable equal to 1 if stop $i \in S$ is upgraded for any BRT line, 0 otherwise
$z_{o,d}^l$	Binary variable equal to 1 if BRT line $l \in L$ covers the demand between $o, d \in S$, 0 otherwise
$\zeta_{o,d}$	Binary variable equal to 1 if the demand between $o, d \in S$ is covered by any BRT line, 0 otherwise

2.1 Problem Setting

We consider a network of stops S and corresponding edges $(i, j) \in S \times S$. Designing a BRT line entails choosing a route, defined by the edges it traverses and the stops it stops at. A BRT line can only use a certain stop if that stop is upgraded. We model the case where BRT lines L are proposed to replace a certain set of existing bus lines L^{up}. Therefore, possible edges for a BRT route are a subset of all edges, for which we introduce the variables $v_{i,j}^l$. Although we represent lines by directed edges in our formulation, a line is assumed to operate both ways, meaning demand in both directions can be satisfied.



(a) Possible BRT line routes **(b) Possible BRT ND**

Figure 1: Example of BRT ND

OD	Demand
$q_{1,2}$	0
$q_{2,3}$	0
$q_{1,3}$	5000
$q_{1,4}$	100
$q_{4,3}$	0

Table 2: OD matrix for the example

Our problem setting is demonstrated using the example network in Figure 1 with the corresponding OD matrix in Table 2. We aim to create two BRT lines A and B, with possible routes shown in Figure 1a. Line A can operate between stops 1 and 3 via stop 2, while line B can operate between stops 1 and 3 directly or via stop 4.

A possible resulting design is shown in Figure 1b. Line A traverses edges $(1,2)$ and $(2,3)$ while line B traverses edges $(1,4)$ and $(4,3)$. Moreover, municipality 2 has no incentive to upgrade stop 2 as there is no demand. Thus, line A only stops at stops 1 and 3, such that $y_1^A = y_3^A = 1$. In contrast, line B stops at every stop it traverses, resulting in $y^B_1 = y^B_3 = y^B_4 = 1$. As there is demand between stops 1 and 4, there is an incentive for municipality 1 to upgrade stop 4. Consequently, stops 1, 3, and 4 are upgraded, so $\gamma_1 = \gamma_3 = \gamma_4 = 1$.

However, municipality 2 could argue that the demand between stops 1 and 4 is small and line B should directly go from 1 to 3. Municipality 1 might agree as it then does not need to invest to upgrade stop 4. This simple example shows the complexity of considering the multiple municipalities when designing a BRT network.

Formally, to incorporate the interests of each municipality $m \in M$, we define a utility function $u_m(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\gamma},\boldsymbol{z},\boldsymbol{\zeta})$, where $\boldsymbol{x} = (x^l_{i,j})^{l\in L}_{i,j\in S}$, $\boldsymbol{y} = (y^l_i)^{l\in L}_{i\in S}$, $\boldsymbol{\gamma} = (\gamma_i)_{i\in S}$, $\boldsymbol{z} = (z^l_{o,d})^{l\in L}_{o,d\in S}$ and $\boldsymbol{\zeta} = (\zeta_{o,d})_{o,d\in S}$. The utility function measures the satisfaction of the municipality with a certain BRT network. The exact utility function can be determined based on the scenario, i.e. the wishes of the municipalities and their budgets. Some examples are provided later in this study. The aim is to maximise the different utilities of the municipalities. To this end, we employ the commonly used weighted sum method to maximise the different utilities. Using this method, we aggregate the utilities by multiplying each of them by a weight w_m and summing over the products, where weights w_m indicate the importance of a municipality, which can, for example, be based on the number of inhabitants or the size of a municipality.

2.2 Mathematical Formulation

The complete $(0,1)$-ILP is then written below.

$$\max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\gamma},\boldsymbol{z},\boldsymbol{\zeta}} \sum\nolimits_{m\in M} w_m \cdot u_m(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\gamma},\boldsymbol{z},\boldsymbol{\zeta}) \tag{1}$$

$$\text{s.t.} \quad x^l_{i,j} \leq v^l_{i,j} \qquad \forall i,j \in S, l \in L \tag{2}$$

$$\sum\nolimits_{i\in S} x^l_{s^l,i} = 1 \qquad \forall l \in L \tag{3}$$

$$\sum\nolimits_{i\in S} x^l_{i,t^l} = 1 \qquad \forall l \in L \tag{4}$$

$$\sum\nolimits_{i\in S} x^l_{i,j} \leq 1 \qquad \forall j \in S, l \in L \tag{5}$$

$$\sum\nolimits_{i\in S} x^l_{j,i} \leq 1 \qquad \forall j \in S, l \in L \tag{6}$$

$$\sum\nolimits_{i\in S} x^l_{i,j} = \sum\nolimits_{i\in S} x^l_{j,i} \qquad \forall j \in S \setminus \{s^l, t^l\}, l \in L \tag{7}$$

$$y^l_i \leq \sum\nolimits_{j\in S} x^l_{i,j} + \sum\nolimits_{j\in S} x^l_{j,i} \qquad \forall i \in S, l \in L \tag{8}$$

$$2 \cdot z^l_{o,d} \geq y^l_o + y^l_d \qquad \forall o,d \in S, l \in L \tag{9}$$

$$z^l_{o,d} \leq y^l_o \qquad \forall o,d \in S, l \in L \tag{10}$$

$$z^l_{o,d} \leq y^l_d \qquad \forall o,d \in S, l \in L \tag{11}$$

$$\gamma_i \geq y^l_i \qquad \forall i \in S, l \in L \tag{12}$$

$$\gamma_i \leq \sum\nolimits_{l\in L} y^l_i \qquad \forall i \in S \tag{13}$$

$$\zeta_{o,d} \geq z^l_{o,d} \qquad \forall o,d \in S, l \in L \tag{14}$$

$$\zeta_{o,d} \leq \sum\nolimits_{l\in L} z^l_{o,d} \qquad \forall o,d \in S \tag{15}$$

$$\sum\nolimits_{i\in S} k^m_i c^{\text{upgr}}_i \gamma_i \leq b_m \qquad \forall m \in M \tag{16}$$

$$\sum\nolimits_{i\in S} y^l_i \leq \#N^l_{\max} \qquad \forall l \in L \tag{17}$$

$$y^l_{s^l} = 1 \qquad \forall l \in L \tag{18}$$

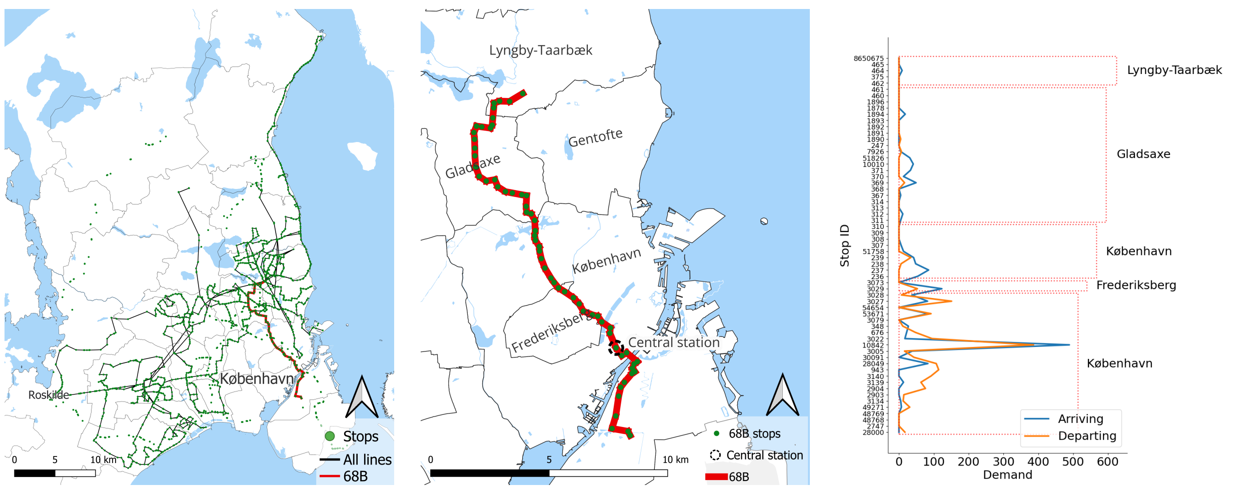
$$y^l_{t^l} = 1 \qquad \forall l \in L \tag{19}$$

The objective function (1) maximises the weighted sum of the municipal utilities. Constraint (2) ensures that a line can only traverse an edge available to it. Constraints (3) and (4) ensure that a line starts and ends at the correct stops. Constraints (5) and (6) state that a stop has at most one incoming and outgoing edge for each line. Constraint (7) ensures flow conservation. Constraint (8) guarantees that a stop can only be upgraded for a line if the line traverses it. Constraints (9 – 11) ensure that a line only covers the demand for a given origin-destination pair if the line contains the origin and destination as stops, without considering transfers. Constraints (12) and (13) state that a stop is upgraded if and only if a BRT line stops there. Constraints (14) and (15) say that the demand between an OD pair is satisfied if and only if it is satisfied for any BRT line. Constraint (16) is the budget constraint for each municipality. Constraint (17) indicates the maximum number of (upgraded) stops for each line. Constraints (18) and (19) ensure the first and last stop for each line are upgraded.

3 Experiments and Results

In this section, we start by describing the experimental setup for the Greater Copenhagen case study. We proceed by describing and motivating three different scenarios and providing results for each of the scenarios.

3.1 Greater Copenhagen Case Study



(a) Bus network in our data **(b) Line 68B with the municipalities** **(c) Demand along the 68B line**

Figure 2: Visualised data supporting the Greater Copenhagen case study

We study the case of upgrading a single existing bus line to a BRT line. Our data consists of a bus network in the Greater Copenhagen area, including stops, existing lines and an OD matrix of the current demand between stops. The stops and lines in our data are visualised in Figure 2a. Our data does not include all lines in the Greater Copenhagen area, but only a subset (Din Offentlige Transport (2024)). We selected line 68B based on the OD demands, as it is the line with the most demand and one of the few lines with demand from and to at least four municipalities. The 68B line, including the involved municipalities and its 61 stops, is visualised in Figure 2b. Figure 2c displays the number of arriving and departing passengers at each stop along the 68B line. In the data we use, no other line covers demand between OD pairs included in the 68B line. Stops in København show the highest demand, while those in Gladsaxe and Lyngby-Taarbæk have relatively low demand.

Formally, the set of lines to upgrade to BRT lines L^{up} only consists of the 68B line. The BRT line set L also only consists of one line, denoted by 68BRT. The involved municipalities, $M^{68\text{BRT}}$, are København, Frederiksberg, Gladsaxe and Lyngby-Taarbæk. We consider all edges for the route of the 68BRT line, i.e. $v_{i,j}^{68\text{BRT}} = 1$ for all edges (i, j) of line 68B. We keep the endpoints $s^{68\text{BRT}}$ and $t^{68\text{BRT}}$ consistent with line 68B.

We also add the constraint that 68BRT always stops at the København Central Station due to its high demand and important location.

Due to the lack of budget data for the 68BRT line, we make some simplifying assumptions. First, we assume the cost of upgrading a stop c_i^{upgr} to be constant. In particular, we assume that $c_i^{\text{upgr}} = 1$ and that the budget of a municipality is equal to the number of stops it can upgrade. Moreover, each municipality has the budget to upgrade each stop from the 68B line such that for $m \in M^{68\text{BRT}}$, the budget is given by $b_m = \sum_{i \in S} k_i^m r_i$.

As mentioned in the introduction, a BRT line often skips stops to increase its speed. For this reason, we consider eight and sixteen for the maximum number of stops $\#N_{\max}^{68\text{BRT}}$. For $\#N_{\max}^{68\text{BRT}} = 16$, there are already more than $\binom{58}{16} \approx 8 \cdot 10^{13}$ different combinations of stops to upgrade. So, although we only consider the current 68B line route for the BRT line, the number of possible solutions is still large.

We consider several scenarios with different utility functions and weights for the municipalities. We mostly focus on the current demand for the 68B line captured by the proposed BRT line. It is reasonable to assume this is also an indicator of potential newly attracted demand. The provided scenarios are also applicable in cases where multiple lines are being considered for upgrades to BRT lines.

First, a base scenario is defined where we maximise total demand for the 68B line satisfied by the proposed BRT line. We achieve this by defining the following utility function for each municipality $m \in M^{68\text{BRT}}$:

$$u_m(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\gamma}, \boldsymbol{z}, \boldsymbol{\zeta}) = \sum_{o \in S} \sum_{d \in S} (k_o^m + k_d^m) \cdot \zeta_{o,d} \cdot q_{o,d}. \tag{20}$$

It is not hard to verify that the objective in (1) reduces to the system-wide demand of the 68B line satisfied by the proposed BRT line when using weights $w_m = 1$.

3.1.1 Scenario 1: Municipal Demand Equity

In scenario 1, we aim for municipal equity regarding the demand. We do this by considering the fraction of the current demand for the 68B line satisfied for each municipality by the proposed BRT line. To this end, we define the utility function for each municipality $m \in M^{68\text{BRT}}$ as

$$u_m(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\gamma}, \boldsymbol{z}, \boldsymbol{\zeta}) = \frac{\sum_{o \in S} \sum_{d \in S} (k_o^m + k_d^m) \cdot \zeta_{o,d} \cdot q_{o,d}}{\sum_{o \in S} \sum_{d \in S} (k_o^m + k_d^m) \cdot \rho_{o,d} \cdot q_{o,d}}. \tag{21}$$

In this scenario, municipal equity is achieved by using equal weights w_m for all municipalities.

3.1.2 Scenario 2: Municipal Demand Equity Considering Budget/Costs

Scenario 2 builds upon scenario 1 by incorporating a cost component into the utility function. Naturally, a municipality prefers to spend as little as possible. Therefore, we add the fraction of the budget remaining after upgrading stops for the proposed BRT line to the utility function. For each municipality $m \in M^{68\text{BRT}}$, we have

$$u_m(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\gamma}, \boldsymbol{z}, \boldsymbol{\zeta}) = \frac{\sum_{o \in S} \sum_{d \in S} (k_o^m + k_d^m) \cdot \zeta_{o,d} \cdot q_{o,d}}{\sum_{o \in S} \sum_{d \in S} (k_o^m + k_d^m) \cdot \rho_{o,d} \cdot q_{o,d}} + \left(1 - \frac{\sum_{i \in S} k_i^m \cdot \gamma_i}{\sum_{i \in S} k_i^m \cdot r_i}\right). \tag{22}$$

It should be noted that we have added the demand equity and budget components together directly for simplicity, but weights could be used based on the importance of both components.

3.1.3 Scenario 3: Municipal Demand Equity Considering Population

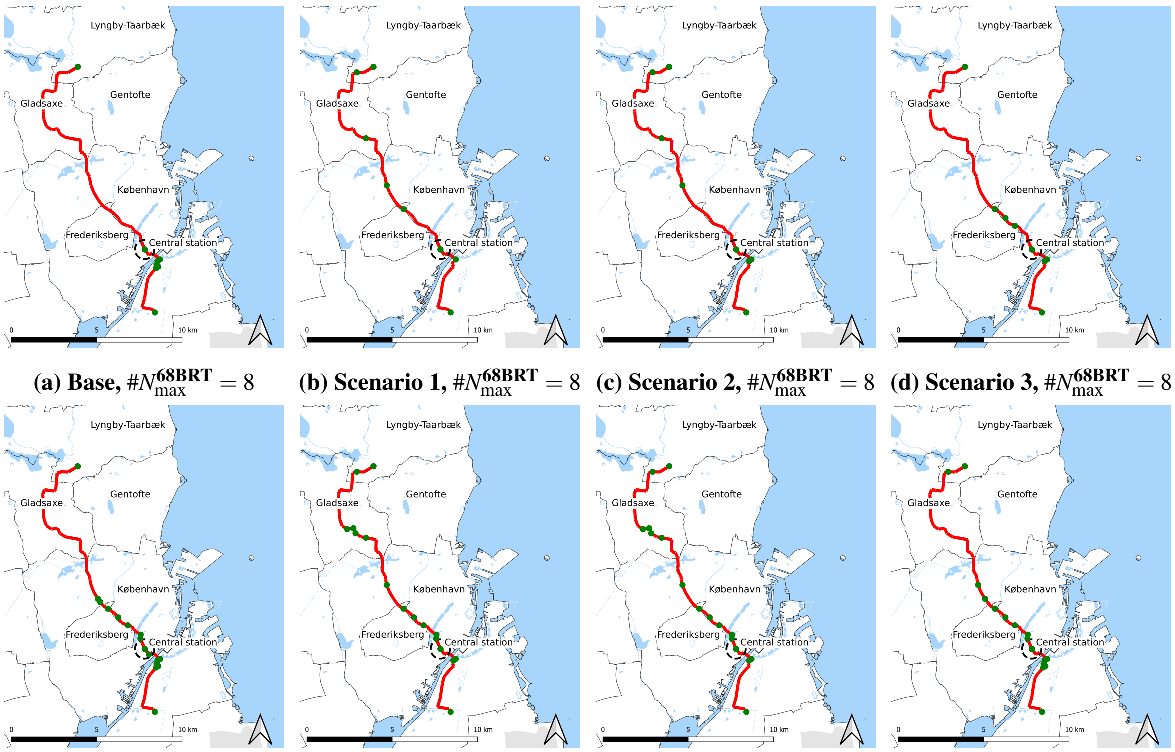
In scenario 3, we extend scenario 1 by using the same utility function (21) but selecting weights proportional to the population in each municipality, which is shown in Table 3 (Statistics Denmark (2024)). This accounts for the larger number of potential passengers in more populated municipalities benefiting from the BRT line.

København	Frederiksberg	Gladsaxe	Lyngby-Taarbæk
667535	106121	71046	58938

Table 3: Number of inhabitants in each municipality of interest on October 1st, 2024

3.2 Results

We determine an optimal solution for each scenario using Gurobi 11.0.3. The solutions are visualised in Figure 3. In the base case, almost all upgraded stops are in København because it has the most demand.



(a) Base, $\#N^{68BRT}_{max} = 8$ **(b) Scenario 1, $\#N^{68BRT}_{max} = 8$** **(c) Scenario 2, $\#N^{68BRT}_{max} = 8$** **(d) Scenario 3, $\#N^{68BRT}_{max} = 8$**

(e) Base, $\#N^{68BRT}_{max} = 16$ **(f) Scenario 1, $\#N^{68BRT}_{max} = 16$** **(g) Scenario 2, $\#N^{68BRT}_{max} = 16$** **(h) Scenario 3, $\#N^{68BRT}_{max} = 16$**

Figure 3: Optimal line stops for the different scenarios

In scenario 1, stops are evenly spread across the different municipalities, which is logical when aiming for municipal demand equity. In scenario 2, no stops in Frederiksberg are upgraded when a maximum of eight stops are considered, i.e. $\#N^{68BRT}_{max} = 8$. This is because line 68B only stops at two stops in Frederiksberg. Thus, upgrading a stop in Frederiksberg would require half of their budget. When $\#N^{68BRT}_{max} = 16$, the gain in demand from upgrading a stop for Frederiksberg outweighs the required budget. This is expected as when more stops are upgraded, the OD demand that can be satisfied by any other upgraded stop increases. Lastly, we see that in scenario 3, again, København receives the most stops as it has the largest population. However, when $\#N^{68BRT}_{max} = 8$, a stop in Frederiksberg is upgraded. For $\#N^{68BRT}_{max} = 16$, an additional stop in Lyngby-Taarbæk is upgraded. The lack of upgrades for stops in Gladsaxe is likely a consequence of the spread of demand across various stops in this municipality, unlike in Lyngby-Taarbæk.

These results show that it is important to consider the interests of different municipalities when designing a BRT line. The base case shows that considering system-wide performance may lead to a BRT line design that is not in the best interests of the municipalities. Moreover, scenario 2 shows that we can capture the trade-off between benefits and costs for a municipality in the utility function. Lastly, scenario 3 shows that weights can be applied to adjust the importance of the involved municipalities.

4 Conclusion and Discussion

In this study, we have introduced a model for determining the routes and stops of BRT systems that maximise the utilities of involved municipalities, using a $(0, 1)-$ILP formulation. We applied our model to a case study of a single line in the Greater Copenhagen region, highlighting the importance of considering different municipalities when selecting stops for a BRT line.

Our formulation can also be used beyond BRT networks to other TND applications, addressing the municipal interests in other infrastructure decisions across multiple regions. For example, regular buses already have infrastructure, and all stops are often used. It is straightforward to generalise our model to this case of regular buses by removing constraints on the budget and maximum number of stops.

A natural extension of our model is the consideration of edge investments to accommodate separate roads for BRT lines, similar to the work of Hoogervorst et al. (2022, 2024). Additionally, incorporating further accessibility metrics, such as travel time, could improve the model. Another interesting extension would be to include the interaction between BRT lines and the surrounding network, making the demand variable. A multi-objective or game-theoretic approach could help better analyse the trade-offs between municipal interests, but would require more data to examine the behaviour of both travellers and municipalities.

A more comprehensive case study is needed to assess the model's applicability to networks with multiple BRT lines and routes. However, the current stop-to-stop OD data limits the evaluation of demand shifts with new routes or stops. Models for transit assignment and (synthetic) generation of travel demand could help estimate these effects. Moreover, our model could be extended to consider transfers between lines. Lastly, with sufficient data, more realistic scenarios for the utility functions, weights and budget could be constructed and studied.

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