

RESEARCH ARTICLE OPEN ACCESS

Multiobjective Transit-Oriented Multimodal Network Design Incorporating Parking Fees

Tygo Nijsten^{1,2}  | Jan-Pieter Dorsman¹  | Michel Mandjes^{1,3}  | Erwin Walraven²  | Maaïke Snelder^{2,4} 

¹Korteweg-De Vries Institute for Mathematics, University of Amsterdam 1098 XG, Amsterdam, the Netherlands | ²Sustainable Urban Mobility and Safety, TNO 2595 DA, The Hague, the Netherlands | ³Mathematical Institute, Leiden University 2300 RA, Leiden, the Netherlands | ⁴Department of Transport & Planning, Delft University of Technology 2628 CN, Delft, the Netherlands

Correspondence: Tygo Nijsten (t.g.c.nijsten@uva.nl)

Received: 26 February 2026 | **Revised:** 7 August 2026 | **Accepted:** 14 August 2026

Academic Editor: Tao Liu

Keywords: evolutionary algorithm | multiobjective optimization | network planning and design | public transport | sustainable transport | transportation planning

ABSTRACT

High car usage in urban areas leads to congestion, pollution, and other negative externalities. While both “carrot” measures (enhancing more sustainable modes of transport) and “stick” measures (discouraging car use) are recognized as essential for sustainable mobility, their combined optimization remains underexplored. This study introduces the Transit Network Design, Frequency, and Parking Setting Problem (TNDFPSP), a novel multiobjective framework that jointly optimizes transit routes, service frequencies, and parking fees in a bimodal context. The optimization problem targets three objectives: minimizing generalized user costs, car usage, and operator deficit, enabling a systematic analysis of trade-offs between user, environmental, and operational goals. We propose an evolutionary algorithm tailored to this problem and evaluate candidate solutions using a multimodal traffic assignment model that explicitly captures car-transit interactions, including parking fees and search times. Experiments on Mandl’s benchmark network show that incorporating parking fees alongside frequencies not only improves all objectives but also enables better trade-off analysis for policy design. As an illustrative estimate based on modal passenger-kilometers, the resulting solutions are associated with an approximately 3.4% lower CO₂ footprint compared to the baseline TNDP, whereas optimizing transit routes and frequencies alone yields only marginal changes. These findings highlight the potential added value of integrated carrot-and-stick strategies for sustainable urban transport planning.

1 | Introduction

The car remains the most popular mode of transport, owing to its convenience and comfort. In the Netherlands, for instance, 50% of all distance traveled in 2022 was by car [1]. However, in many urban areas, such high car usage leads to substantial road congestion and is accompanied by several negative externalities, including noise pollution, greenhouse gas emissions, and an increased number of traffic accidents. To mitigate these impacts, promoting a shift towards more sustainable modes of transport, such as public transport, is often desirable. Measures aimed at

achieving this shift can generally be classified into two categories: actions that encourage the use of other modes like public transport and actions that discourage car use, commonly referred to as carrot measures and stick measures, respectively. In this study, we aim to combine the optimization of the two types of measures to support a transition towards greater public transport usage. We briefly review typical carrot and stick measures below.

A logical carrot measure is to improve the design and planning of transit networks. This process typically involves five sequential stages: (1) network route design, (2) frequency setting, (3)

This is an open access article under the terms of the [Creative Commons Attribution](https://creativecommons.org/licenses/by/4.0/) License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

Copyright © 2026 Tygo Nijsten et al. *Journal of Advanced Transportation* published by John Wiley & Sons Ltd.

timetable development, (4) vehicle scheduling, and (5) driver scheduling [2]. Optimizing all five stages simultaneously is infeasible, as each stage is NP-hard on its own [3]. Consequently, much of the existing literature focuses on the first two stages, which have the greatest impact on passenger service quality and operator costs. This focus has given rise to two well-known problems: the Transit Network Design Problem (TNDP) and the Frequency Setting Problem (FSP). The TNDP concerns optimizing public transport line routes, while the FSP involves determining the service frequencies of these routes. Traditionally, these problems have been solved sequentially. However, in recent years, research interest has grown in combining the two phases, leading to the formulation of the Transit Network Design and FSP (TNDFSP) [4].

Obvious measures of the stick type include congestion tolls and parking prices. It is known that these interventions regarding cars can influence transit ridership, concerning, among other things, route choice and departure time [5]. Both theory and practice have shown that the ex-ante majority is against the introduction of road pricing [6]. Parking fees, on the other hand, are already enforced in many urban areas. Moreover, the technology needed to implement congestion tolls is considerably more complicated than that of introducing parking prices [7]. Thus, parking pricing is easier to introduce in practice and is therefore the more relevant stick measure to consider.

In the existing literature, multiple gaps can be identified. First, to our best knowledge, no research has yet incorporated the optimization of parking fees, or any other stick measure for that matter, in the TNDFSP. When considering the design of both the transit and car network at the same time, this is often referred to as a multimodal network design problem (MMNDP) [8]. The literature suggests that combining measures to encourage a shift towards more sustainable modes of transport and discourage car usage could be more effective than either in isolation [9]. Nevertheless, the literature on the interaction between transit, parking, and mode shift is still limited [10].

Second, increasing the modal shift towards public transport as a sustainability goal is generally not considered an explicit objective when solving the TNDFSP. Most studies on the TNDFSP only consider user and/or operator costs as objectives [11]. However, considering the modal shift towards public transport as an objective might come at a price. Except for very short trips (< 3 km), trips made by public transport generally take longer than trips made by car [12]. Furthermore, improving the transit network can be expensive, and transit operators might have limited budgets. Therefore, it is important to take into account the multiple conflicting objectives.

To deal with such conflicting objectives, they are often incorporated into a single objective, where each objective has a certain weight. However, the weights are usually determined by a policymaker and introduce uncertainty, as it is difficult to select them accurately [13]. Multiobjective optimization considers separate objectives simultaneously. Typically, due to these trade-offs, no single solution exists that optimizes all objectives simultaneously. Instead, multiobjective optimization usually aims to find a set of solutions that cannot be improved in any objective without a sacrifice in another objective, known as the Pareto front. Often, evolutionary algorithms are used to solve such multiobjective problems [14]. These methods explore a large solution space and therefore require repeated evaluations of

candidate solutions, which makes computational efficiency an important consideration.

In this paper, we address these gaps by introducing the Transit Network Design, Frequency and Parking Setting Problem (TNDFPSP), a multiobjective MMNDP that jointly optimizes transit network design and parking pricing strategies. Specifically, we consider three objectives: (1) minimizing generalized user costs, (2) minimizing the number of car users, and (3) minimizing operator deficit. The multiobjective approach allows a systematic analysis of the economic, environmental, and operational trade-offs, providing a framework for policymakers to evaluate the combined effect of transit improvements and parking policies on the mobility system. This paper makes the following contributions:

- **Joint optimization of carrot and stick measures to reduce car usage.** We introduce the TNDFPSP, a multi-objective extension of the TNDFSP that simultaneously optimizes both carrot measures (transit routes and frequencies) and stick measures (parking fees) within a unified framework. Our approach explicitly captures the dependencies between transit and parking decisions, which strongly influence modal split and the three objectives. While some prior studies have considered aspects of combining carrot and stick measures, to the best of our knowledge, this is the first to systematically and jointly optimize transit network design, frequency setting, and parking pricing in a multiobjective framework.
- **Integrated multimodal modeling.** Our framework combines a road user-equilibrium model, a frequency-based public transport assignment, and a nested logit mode-choice model. By explicitly including parking capacity and fees, we realistically represent interactions between car and transit modes under both transit and parking policies.
- **Tailored evolutionary solution method.** We apply an evolutionary algorithm adapted to handle parking fees as decision variables, alongside transit routes and frequencies. In addition to a repair operator, we propose an expansion operator that exploits the problem structure to extend the Pareto front without significant computational effort.
- **Empirical demonstration and policy relevance.** Using Mandl's benchmark network, we show that joint optimization of carrot and stick measures in a bimodal setting can reduce car usage and, as a consequence, CO₂ emissions, while still balancing generalized user costs and operator deficit (i.e., accounting for the impact of mode shift on fare revenue and operating costs). Sensitivity analyses further highlight the robustness and practical value of our approach.

The paper is structured as follows. In Section 2, we present an overview of the related literature and detail how our study contributes to it. The framework to model transit and car traffic, as well as the proposed TNDFPSP, is addressed in Section 3. In Section 4, we present our proposed evolutionary algorithm. We describe the experimental design to test our algorithm in Section 5. In Section 6, we first compare our algorithm's performance with existing methods for TNDPs without optimizing frequencies and parking fees. Consequently, we show the benefits of combining the TNDP with the optimization of frequencies and, more importantly, the parking fees by solving the presented TNDFPSP. We finish this section with a sensitivity analysis regarding some of the parameters we introduce

throughout this paper. Finally, conclusions and possible future research directions are presented in the last section.

2 | Related Literature

In this section, we review the literature related to the proposed TNDFPSP. We start by discussing literature on multimodal network design. We then focus on transit network design in particular. We proceed by reviewing the literature on traffic assignment models needed to study multimodal networks. Finally, we examine existing solution methods for variants of the TNDFPSP.

2.1 | Multimodal Network Design

Recent studies have advanced multimodal network design in several ways, for example, by optimizing road capacities in bi-modal networks [15, 16], jointly addressing transit network design and road expansions [17], or focusing on frequency optimization in bi-modal settings [18]. Other works integrate on-demand or shared mobility with public transport [19–24] or investigate park-and-ride (P&R) facilities [25–29]. While these works aim to improve public transport and other modes of transport, so-called carrot measures, they do not incorporate stick measures to discourage private car use.

A small body of studies has started to incorporate stick measures into transit network design and planning. For example, some studies partially integrate parking into the planning process, such as by optimizing parking fees with a fixed transit supply [30] or jointly optimizing transit fares and parking fees [31]. However, these do not consider route and frequency design as decision variables, the core decision variables in transit network design. Loder et al. [32] explore a scenario with reduced parking in a multimodal context but do not regard parking as a decision variable.

To our knowledge, no prior work has jointly optimized parking policies and core transit network design variables within a unified framework. Most multimodal network design studies treat parking as a fixed input rather than a decision variable interacting with transit design. Our work addresses this gap by jointly optimizing transit routes, frequencies, and parking fees in a multiobjective framework, enabling explicit analysis of the combined effects of optimizing these carrot and stick measures, an aspect missing from previous literature.

2.2 | Objectives and Multimodality in Transit Network Design

Transit network design is typically studied in a unimodal framework, focusing primarily on minimizing user costs and, less frequently, operator costs [11]. In unimodal settings, considering operator costs is sufficient, but in bimodal or multimodal contexts, operator profit becomes important to consider, as fare revenue depends on transit ridership, which itself is influenced by competition with other modes. Therefore, we consider the operator deficit as an objective instead.

Although some recent works consider additional objectives such as equity [33, 34] or emissions [35–37], these remain limited. Even when emissions are included, the number of car users is typically not an explicit objective, resulting in solutions that favor

car use due to user cost minimization, corresponding to higher levels of emissions.

A meaningful analysis of modal shift requires a bimodal or multimodal approach, where both transit and private car are modeled, and transit demand is variable. While some studies add walking or a generic competing mode to the network [38–40], only a few explicitly model cars as a substitute for transit [33–35, 41]. Among these, parking is often ignored and not treated as a decision variable. While Almasi et al. [41] do incorporate parking fees in their model, the fees are treated as fixed parameters, and park search time is omitted, which is a substantial component of total travel time in urban areas [42].

Our TNDFPSP formulation addresses these gaps by introducing the number of car users as an objective, alongside user costs and operator deficit, and by explicitly incorporating parking fees as decision variables and park search time in the network design. This enables direct study of the trade-offs between reducing car usage, maintaining acceptable travel times and user costs, and ensuring financial sustainability for operators, providing a more realistic and policy-relevant analysis on modal shift.

Furthermore, whereas most previous studies compute frequencies based on required service [11], our framework incorporates them as explicit decision variables. This allows us to capture the feedback between service provision and demand, which is essential for realistic modal shift analysis. Notably, in such a setting, reducing service does not necessarily decrease operator deficit, as lower costs may be canceled by reduced fare revenue when demand shifts away from public transport.

2.3 | Traffic Assignment Models

Traffic assignment models determine how travelers distribute themselves across available routes and modes. For car traffic, most studies assume selfish drivers who minimize their own travel costs, leading to the Wardrop equilibrium [43], typically computed using algorithms such as Frank–Wolfe [44]. We adopt this approach because road congestion strongly influences generalized costs and route choice, making it necessary for realistic multimodal modeling.

For public transport, assignment models range from simple shortest-path assumptions [45] to frequency-based approaches such as optimal strategies and frequency-share models [46, 47] and to more advanced capacity-constrained schedule-based models [48, 49].

Frequency-based models, which account for schedule uncertainty without requiring full timetables, are most common in TNDFPSP research [11, 50]. Such models are particularly appropriate when service frequencies are decision variables, as travelers typically make decisions based on service frequencies and expected waiting times rather than exact vehicle departure times. Simulation-based frameworks (e.g., MATSim [51]) and learning-based approaches [52–54] offer greater behavioral realism but require significant data and are computationally intensive, so they are less suited for repeated evaluations in large-scale optimization.

Because multiobjective network design requires evaluating many candidate solutions, highly detailed simulations or capacity-constrained models are often computationally restrictive. We therefore adopt a frequency-share model without explicit crowding effects, balancing behavioral realism and

computational efficiency, a standard approach in TNDFSP studies [11]. In contrast, car congestion is explicitly modeled, as road capacity has a direct and significant impact on travel times and modal split.

2.4 | Solution Method

The TNDFSP is an NP-hard problem, and our extension to include parking fees (TNDFPSP) inherits this complexity. Exact methods are only feasible for highly simplified instances, so most studies rely on metaheuristics.

Among these, multiobjective Genetic Algorithms (GAs) are most common, including the Alternating Objective GA (AOGA) introduced by Arbex and da Cunha [55], a similar approach presented by Chew et al. [56], the route-construction framework of Owais et al. [57], and the hierarchical multiobjective GA of Owais and Osman [58]. The Nondominated Sorting GA II (NSGA-II) proposed by Deb et al. [59] has become the standard in multiobjective TNDFSP literature due to its strong balance between convergence and diversity, straightforward implementation, and reproducibility [11]. NSGA-II variants have been widely applied to solve the TNDFSP [34, 36, 41, 45, 60, 61].

Alternative metaheuristics, such as particle swarm optimization (PSO), used by Kechagiopoulos and Belgiannis [62] and Iliopoulou and Kepaptsoglou [63], as well as artificial fish swarm algorithms, applied by Liu et al. [64], have also been explored to solve the TNDFSP but yield limited Pareto fronts or focus on aggregated objectives rather than explicit trade-offs.

More advanced evolutionary algorithms, such as NSGA-III [65] and MOEA/D [66], are designed to solve many-objective problems but require complicated parameterization (e.g., reference-point calibration, weight vector selection) [67–70]. Liang et al. [71] proposed the MOEA-OSD, which is based on the MOEA/D framework, to solve a bi-objective TNDFPSP. Similarly, Aksoy and Alver [72] apply MODEA, which uses the NSGA-III selection operator. However, both methods are designed and validated for bi-objective problems and would require significant adaptation and calibration to be applicable in our three-objective case.

As the field continues to evolve, recent work has begun to explore deep reinforcement learning and other machine learning approaches for solving TNDFPs [73, 74]. This trend is also highlighted in recent reviews, which identify AI- and machine-learning-based approaches as a promising yet underexplored direction in transit network design research [75]. While these methods show promise, as noted by Nnene et al. [74], learning-based approaches are highly data-intensive and computationally expensive, challenges that are even more pronounced in multimodal settings and in evolutionary search that evaluates many candidate designs.

Given these considerations, NSGA-II offers the best balance of performance, simplicity, and comparability for our three-objective, discrete-variable TNDFPSP. Unlike more advanced algorithms, NSGA-II does not require complex parametrization and is well-established in the literature. For these reasons, we adopt NSGA-II as the core of our solution method.

3 | Mathematical Formulation

In this section, we formalize the proposed TNDFPSP, which aims to minimize user costs, the number of car users, and the operator

deficit by optimizing transit routes, frequencies, and parking fees. To quantify these objectives, we develop a multimodal traffic assignment model that extends existing approaches to jointly evaluate transit network design and parking pricing policies. Figure 1 illustrates the overall scheme. We begin by defining the multimodal network (Section 3.1), including car and public transport subnetworks, with parking fees incorporated in the car network and transit routes and frequencies in the public transport network. We then describe the generalized costs for both modes (Section 3.2), followed by the multimodal assignment model (Section 3.3), which consists of car assignment, public transport assignment, and modal split. Section 3.4 presents the mathematical formulation of the objectives, and Section 3.5 states the formal optimization problem.

3.1 | Network Definition

The mathematical formulation relies on a directed graph $G = (\mathcal{N}, \mathcal{A})$ that represents a physical multimodal transportation network. The network is divided into two subnetworks: a car network and a public transport network. Later in this section, we provide the details for both of these sub-networks. However, before doing so, we need to define some generic concepts.

We define a set of zonal centroids as a subset of all the nodes, i.e., $\mathcal{Z} \subseteq \mathcal{N}$. A zonal centroid $z \in \mathcal{Z}$ represents a point within a zone that serves as an origin or destination for people traveling from or to this zone. A combination of an origin $o \in \mathcal{Z}$ and a destination $d \in \mathcal{Z}$ is called an origin–destination (OD) pair. The set of all OD pairs is denoted by $\mathcal{S} \subseteq \mathcal{Z} \times \mathcal{Z}$. Each OD pair $(o, d) \in \mathcal{S}$ is associated with a demand $q^{o,d}$ specifying the number of travelers that want to travel from o to d . Furthermore, each link $a \in \mathcal{A}$ has a corresponding length ℓ_a . We denote the flow on a link by x_a and the time it takes to traverse this link by a delay function $D_a(x_a)$. We now detail the car and public transport networks.

The car network is described by the graph. The node set $\mathcal{N}^c$ consists of all road intersections and parking nodes, which are

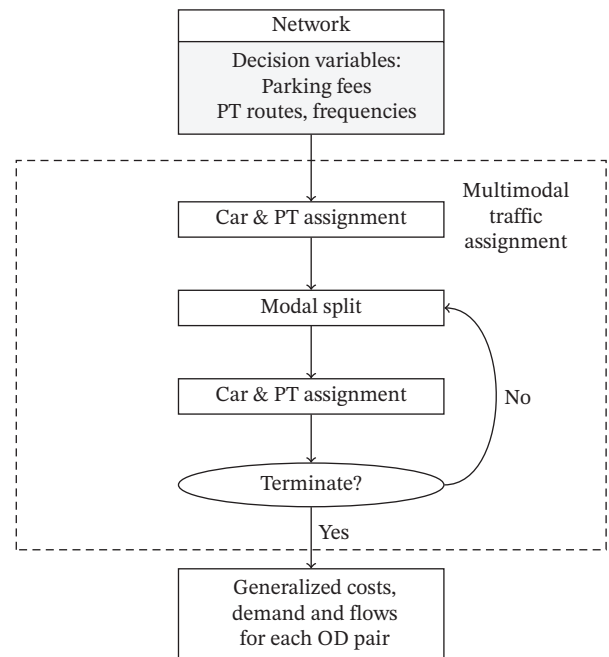


FIGURE 1 | Scheme of multimodal traffic assignment.

denoted by $\mathcal{N}_r$ and $\mathcal{N}_p$, respectively. The link set $\mathcal{A}^c$ consists of road, parking, and walk links, denoted by $\mathcal{A}_r$, $\mathcal{A}_p$, and $\mathcal{A}_w$, respectively. Each parking node and link corresponds to a road intersection. Parking nodes represent a parking facility at that road intersection, while parking links represent the time it takes to search for a parking space. In our model, a user can walk between parking nodes and road intersections as long as they are within a certain distance limit, which we denote by $\ell_{\max}^c$. In other words,

$$\mathcal{A}_w^c = \{(i, j) \in \mathcal{A}_w \mid i, j \in \mathcal{N}_p \times \mathcal{Z}, \ell_{(i, j)} \leq \ell_{\max}^c\}. \quad (1)$$

For each type of link, we define corresponding delay functions. As the delay functions on road links, we use the BPR functions as introduced in [76]:

$$D_a(x_a) = t_a \cdot \left(1 + \alpha_a \left(\frac{x_a}{c_a}\right)^{\beta_a}\right), \quad (2)$$

where t_a is the free flow travel time, c_a is the capacity of link a and $\alpha_a > 0$ and $\beta_a > 0$ are edge-specific constants. Ryan [76] suggests taking $\alpha_a = 0.15$ and $\beta_a = 4$ for American highways, but the first parameter is usually larger for urban areas; for example, a value like $\alpha_a = 2$ is typically used. As proposed by Lam et al. [77], BPR functions can also be used for parking links, but typically with higher values of α_a and β_a . In the case of parking links, capacity refers to the number of parking spots at the corresponding parking node, while flow refers to the number of cars parking there. We suppose that the delay function on walking links is constant and independent of the flow. Furthermore, we assume that a traveler using the car can depart immediately, i.e., access time is zero. A sequence of connected links $a \in \mathcal{A}^c$ between an origin and destination centroid is called a route. We denote the set of all (car) routes by $\mathcal{R}^c$. An important assumption we make throughout this paper is that every OD pair is connected in the car network. Thus, there is at least one (car) route between each OD pair.

The transit network is characterized by the subgraph $\mathcal{G}^t = (\mathcal{N}^t, \mathcal{A}^t)$. The node set $\mathcal{N}^t$ can be seen as a set of transit stops. The link set $\mathcal{A}^t$ can be further divided into transit, access, egress, and transfer links, which we will specify now. A transit line l is described by a sequence of consecutive nodes $\mathcal{N}_l^t \subseteq \mathcal{N}^t$ and a set of links $\mathcal{A}_l^t$ connecting them. The transit links then consist of the links of all lines combined. Moreover, each line is equipped with a frequency f_l and a speed v_l . The set of all transit lines is denoted by $\mathcal{L}$. Walking and bicycle links function as access and egress to and from public transport stops if the distance to a zonal centroid is within a particular limit $\ell_{\max}^{ae}$. Furthermore, besides transfers at the same stop, walking links between different stops can be used to transfer between transit lines, again within some distance limit $\ell_{\max}^s$. Formally, we have $\mathcal{A}^t = \mathcal{A}_{pt} \cup \mathcal{A}_a^t \cup \mathcal{A}_e^t \cup \mathcal{A}_s^t$, where

- $\mathcal{A}_{pt} = \cup_{l \in \mathcal{L}} \mathcal{A}_l^t$ is the transit link set
- $\mathcal{A}_a^t = \{(i, j) \in \mathcal{A}_w \cup \mathcal{A}_b \mid (i, j) \in \mathcal{Z} \times \mathcal{N}^t, \ell_{(i, j)} \leq \ell_{\max}^{ae}\}$ is the access link set
- $\mathcal{A}_e^t = \{(i, j) \in \mathcal{A}_w \cup \mathcal{A}_b \mid (i, j) \in \mathcal{N}^t \times \mathcal{Z}, \ell_{(i, j)} \leq \ell_{\max}^{ae}\}$ is the egress link set

- $\mathcal{A}_s^t = \{(i, j) \in \mathcal{A}_w \mid (i, j) \in \mathcal{N}^t \times \mathcal{N}^t, \ell_{(i, j)} \leq \ell_{\max}^s\}$ is the walk transfer link set.

Note that we consider both walking and cycling as access and egress modes, but only walking as a transfer mode. We assume the delay function for transit, walking, and bicycle links to be constant. Similar to the car route set, there is also the set of all transit routes, $\mathcal{R}^t$.

Putting the two subnetworks together, the node set $\mathcal{N} = \mathcal{N}^c \cup \mathcal{N}^t \cup \mathcal{Z}$ consists of the nodes defined for the car network, the nodes defined for the transit network, and the zonal centroids. The link set $\mathcal{A} = \mathcal{A}^c \cup \mathcal{A}^t$ consists of the links defined for the car network and the links defined for the transit network.

Although the car and public transport subnetworks are defined separately, they are integrated through the travel demand and mode choice process. Each OD pair is represented in both subnetworks, and the assignment of flows to each network is determined by the modal split procedure, which is described in detail in Section 3.3.3. This approach reflects a long-term, strategic network design perspective: we do not model dynamic or real-time transitions of individual travelers between networks. Instead, the integration occurs at the aggregate level, where changes in network design and policy (such as parking fees, transit routes, and frequencies) influence the distribution of traffic across modes.

3.2 | Generalized Costs

To describe traveler behavior, we specify the generalized costs for each route, both for car and public transport travel between OD pairs $(o, d) \in \mathcal{S}$. Generalized costs mainly comprise time and monetary components, describing how travelers perceive a trip.

For car travelers, a trip consists of driving to a parking node d' , searching for parking, and walking to the destination centroid d . The generalized costs consist of the following time components: (1) in-vehicle travel time, (2) park search time, and (3) egress walk time, as well as the cost-distance component, fuel costs, and the cost component, parking fee.

Formally, the generalized costs for a car driver traveling between OD pair (o, d) using some route $r \in \mathcal{R}^c$ are given by

$$c_{car, r}^{o, d} = \mu_{car} + \text{VoT}_{car} \cdot T_{car, r}^{o, d'} + \text{VoT}_{search} \cdot T_{search}^{d'} + \text{VoT}_{walk} \cdot T_{walk}^{d', d} + c_{\$} \cdot d_{car, r}^{o, d'} + p^{d'} \cdot T_{park}, \quad (3)$$

where

- μ_{car} is a car-specific constant
- VoT_i for $i \in \{\text{car}, \text{search}, \text{walk}\}$ are some coefficients based on the value of time (VoT) of travelers
- $T_{car, r}^{o, d'}$ is the in-vehicle travel time from o to d' using route r
- $T_{search}^{d'}$ is the park search time at d'
- $T_{walk}^{d', d}$ is the walking time from d' to zonal centroid d
- $c_{\$}$ is a coefficient for the fuel price per kilometer
- $d_{car, r}^{o, d'}$ is the in-vehicle distance traveled from o to d' using route r
- $p^{d'}$ is the parking fee at node d'

- T_{park} is the average parking duration, which we assume to be constant.

For transit users, generalized costs account for both access and egress modes. A trip includes access from origin o to a stop o' , in-vehicle travel to stop d' , and egress to destination d , possibly with transfers. Transfers incur a penalty, and waiting time is included. The generalized costs given access and egress modes are thus a combination of the following time components: (1) access travel time, (2) in-vehicle travel time, (3) walk transfer time, (4) waiting time, and (5) egress travel time, as well as the cost-distance component transit fee and a transfer penalty if transfers are made. Formally, the generalized costs for a transit user traveling between OD pair (o, d) , while using some route and choosing access mode $a \in \{\text{walk, cycle}\}$ as well as egress mode $e \in \{\text{walk, cycle}\}$, are given by

$$\begin{aligned} c_{\text{transit},a,e,r}^{o,d} = & \mu_{a,e} + \text{VoT}_a \cdot T_a^{o,o'} + \text{VoT}_{\text{transit}} \cdot T_{\text{transit},r}^{o',d'} \\ & + \text{VoT}_{\text{walk}} \cdot T_{\text{wt},r}^{o',d'} + \text{VoT}_e \cdot T_e^{d',d} + \\ & \text{VoT}_{\text{wait}} \cdot T_{\text{wait},r}^{o',d'} + t_s(d_{t,r}^{o',d'}) + \lambda \cdot NT_r^{o',d'}, \end{aligned} \quad (4)$$

where

- $\mu_{a,e}$ is a transit- and access/egress-specific constant
- VoT_i for $i \in \{\text{transit, walk, cycle, wait}\}$ are some coefficients based on the VoT of travelers
- $T_a^{o,o'}$ is the access travel time from centroid o to stop o' using route r and access mode a
- $T_{\text{transit},r}^{o',d'}$ is the in-vehicle transit travel time from stop o' to stop d' using route r
- $T_{\text{wt},r}^{o',d'}$ is the time spent walking for transfers when traveling from o' to d' using route r
- $T_e^{d',d}$ is the egress travel time from stop d' to centroid d using route r and egress mode e
- $T_{\text{wait},r}^{o',d'}$ is the time spent waiting when traveling from o' to d' using route r
- t is a function of the in-vehicle transit kilometers for the transit fee
- $d_{t,r}^{o',d'}$ is the in-vehicle distance traveled from o' to d' using route r
- λ is a coefficient for the transfer penalty
- $NT_r^{o',d'}$ is the number of transfers when traveling from o' to d' using route r

3.3 | Multimodal Traffic Assignment

The multimodal traffic assignment model in this study consists of three main components: car assignment, public transport assignment, and modal split (see Figure 1). Each component is chosen to balance behavioral realism and computational efficiency, which is essential for evaluating many candidate solutions in multiobjective optimization. Car traffic is assigned using a user equilibrium (UE) model based on Wardrop's principle, capturing congestion effects. Public transport assignment uses

a frequency-share model based on the Zenith algorithm [25], which realistically models passenger route choice and schedule uncertainty. These assignment approaches reflect the different characteristics of the two modes: private-car users are modeled according to a user-equilibrium principle, whereas public transport users are represented through a frequency-based assignment model that explicitly accounts for waiting-time effects and schedule uncertainty. We do not include explicit vehicle capacity constraints or crowding effects in the transit assignment, as road congestion has a more direct and continuous impact on travel times and mode choice, while crowding in public transport only becomes significant at full capacity [78]. This modeling choice ensures computational tractability while capturing the dominant effects relevant for modal split and policy analysis.

While our car and transit assignment components are based on established algorithms, the Frank-Wolfe method for UE and the Zenith algorithm as an extension of Spiess' optimal strategy model, our framework extends these by directly integrating parking fees and park search time into the multimodal traffic assignment. By doing so, we can effectively analyze the effects of parking policies on the behavior of car users and the modal split between cars and transit. The car and transit assignments are linked through an iterative modal split process, where updated flows and costs in one mode affect demand and costs in the other. These adaptations allow us to capture key interactions between parking policy, transit network design, and multimodal route choice while maintaining computational tractability.

3.3.1 | Car Assignment

For the car assignment, we assume Wardrop's principle [43]: users minimize their generalized costs, leading to a UE where no traveler can improve their cost by changing routes. We refer to the generalized costs for traveling by car between an OD pair $(o, d) \in S$ as $c_{\text{car}}^{o,d}$. This UE requires the delay functions to be (1) strictly increasing (greater congestion leads to more delay) and (2) continuously differentiable (ensuring smoothness), which both hold for our chosen types of delay functions. We approximate the UE using the Frank-Wolfe algorithm [44]. In each iteration, the current link flows are used to compute the travel costs, after which the shortest paths between all OD pairs are determined using Dijkstra's algorithm. An all-or-nothing assignment is then performed, assigning all demand for each OD pair to its current shortest path. The new flows are updated by averaging the previous flows and the all-or-nothing flows, following the standard Frank-Wolfe step size. This iterative process moves the solution closer to the UE. For computational efficiency, we run the algorithm for a fixed number of iterations, nc_{max} , as specified in Section 5.

3.3.2 | Transit Assignment

For the transit assignment, we use the Zenith algorithm [79], an extension of the optimal strategy model [46]. Both methods work by traversing routes backwards from the destination to the origin, similar to Dijkstra's shortest path algorithm. The main differences between the optimal strategy method and the Zenith algorithm, though, are that for the Zenith algorithm, access and egress are included, and that generalized costs play a role in the route choice for users. In particular, in the optimal strategy

method, it is assumed that the traveler boards the first transit vehicle to arrive and has no prior information. However, with the current technology, it is reasonable to believe that many travelers know of the generalized costs for different routes. This introduces a bias towards routes with lower generalized costs. Therefore, considering generalized costs in the route choice for users makes the Zenith algorithm more realistic.

Now, let NT be a counter for the number of transfers considered and let $NT_{\max}$ be the maximum number of transfers considered. Then the transit assignment algorithm that we employ can be summarized as follows:

- For each OD pair $(o, d) \in S$ and for each access and egress mode $a, e \in \{\text{walk}, \text{cycle}\}$:
 - Select possible access and egress stops based on certain criteria, such as a maximum distance.
 - Set the generalized costs from each egress stop to the destination equal to the travel times using egress mode e and set costs for all other stops equal to infinity.
 - Initialize the stops to be processed as the set of egress stops.
 - For NT from 0 to $NT_{\max}$:
 - Propagate backwards from each stop in the set of stops to be processed along all lines that serve it.
 - A line choice model is applied to determine the fraction of users choosing each line at a stop. This fraction depends on the frequencies of each line and the generalized costs to reach the destination using that line.
 - Add stops to the stop set if they reduce the generalized costs.
 - Add stops that can be used for a walking transfer to the stop set.
 - Update generalized costs.
 - For each access stop, we now know the generalized costs for reaching the destination.
 - A stop choice model is applied to determine the fraction of users choosing each access stop.

For both the line and stop choice models, a scale parameter indicates the extent to which the choice is sensitive to changes in generalized costs. The exact formulation of the line and stop choice models can be found in the work of Brands et al. [79]. In the remainder of this paper, we refer to these parameters as τ_{line} and τ_{stop} . The transit assignment algorithm helps us determine the generalized costs for each access and egress mode on each OD pair, $c_{\text{transit},a,e}^{o,d}$, demand for each access and egress mode on each OD pair, $q_{\text{transit},a,e}^{o,d}$ and the passenger flow for each transit link.

3.3.3 | Modal Split

We assume that the total demand for each OD pair is fixed, but that the split between car and transit is determined by a nested logit model [80], with two nests: the first for the choice between car and public transport, and the second for the choice between different access and egress combinations for public transport. This means that car and transit demand on their own are variable.

To analyze the split in only the first nest, we consider the composite generalized costs for traveling by transit between OD pair (o, d) :

$$c_{\text{transit}}^{o,d} = -\tau \cdot \log \left(\sum_{a,e \in \{\text{walk}, \text{cycle}\}} \exp \left(-\frac{1}{\tau} \cdot c_{\text{transit},a,e}^{o,d} \right) \right). \quad (5)$$

The parameter $0 < \tau \leq 1$ indicates the dissimilarity between the access and egress combinations. Given total demand $q^{o,d}$ and the (composite) generalized costs $c_{\text{car}}^{o,d}$ and $c_{\text{transit}}^{o,d}$ for OD pair (o, d) , the demand for each mode $m \in \{\text{car}, \text{transit}\}$ is given by

$$q_m^{o,d} = q^{o,d} \cdot \frac{\exp(-\theta c_m^{o,d})}{\exp(-\theta c_{\text{car}}^{o,d}) + \exp(-\theta c_{\text{transit}}^{o,d})}, \quad (6)$$

where θ is a logit scale parameter for the choice between car and transit. This scale parameter indicates the sensitivity of the mode choices to changes in generalized costs. Often, but not always, this scale parameter in the nested logit model is normalized, i.e., $\theta = 1$.

The multimodal assignment process is performed iteratively. In the first iteration, the network is empty, and travel costs for car users are low, resulting in a higher initial demand for the car. As flows are assigned and the network becomes loaded, congestion increases travel times for car users, which in turn shifts some demand from car to transit in subsequent iterations. This process continues, with updated costs and modal shares, until the modal split stabilizes or a maximum number of iterations is reached. For computational reasons, we impose a maximum of two loops, which preliminary analyses showed to be sufficient for our applications.

Although the nested logit model is derived from individual choice theory, it is applied here at the aggregate OD-pair level. Given the strategic nature of the planning problems considered, the focus is on estimating network-wide changes in travel demand and modal split rather than reproducing individual travel choices. This level of aggregation is consistent with the intended application of the model and yields demand estimates suitable for traffic assignment and network optimization.

In our model, in-vehicle transit travel times are not affected by car congestion; the networks are treated separately, with congestion only considered for car traffic. This reflects both empirical and computational considerations: road congestion directly impacts car travel times, while transit crowding only affects travel times at full capacity.

3.4 | Objective Functions

We simultaneously optimize three objectives: (1) generalized user costs, (2) the number of car users, and (3) operator deficit. Rather than focusing on a single objective, our approach systematically explores trade-offs between these goals, aiming to find the Pareto front, the set of solutions for which no objective can be improved without sacrificing at least one other, that balance user, operator, and environmental considerations.

To obtain the generalized user costs, we first compute the total generalized costs for each OD pair o, d by adding the generalized costs for each mode. For the car users, this is the product of the generalized costs $c_{\text{car}}^{o,d}$ for an individual and the demand $q_{\text{car}}^{o,d}$. For transit, we first look at the generalized costs for an access and egress combination a, e by multiplying the generalized costs $c_{\text{transit},a,e}^{o,d}$ for an individual and the

demand $q_{\text{transit},a,e}^{o,d}$. We then add the generalized costs for each of these combinations together to obtain the generalized costs for transit users for OD pair o, d . Finally, we add the generalized costs for each OD pair together to obtain the total generalized user costs. Formally, the generalized user costs are given by

$$\sum_{(o,d) \in S} \left(c_{\text{car}}^{o,d} \cdot q_{\text{car}}^{o,d} + \sum_{a,e \in \{\text{walk}, \text{cycle}\}} c_{\text{transit},a,e}^{o,d} \cdot q_{\text{transit},a,e}^{o,d} \right). \quad (7)$$

The number of car users is given by the sum of the demand for traveling by car for each OD pair:

$$\sum_{(o,d) \in S} q_{\text{car}}^{o,d}. \quad (8)$$

In most studies on the TNDFSP, when operator costs are considered, they are typically modeled as a function of the total fleet size [11]. However, we argue that in a bimodal setting, revenues from passenger fares should also be taken into account, as higher transit ridership directly translates into higher income. Therefore, we consider the operator deficit, defined as total operating costs minus fare revenues.

The operator's revenue consists of the fee paid by transit users. We assume that there is only one operator and that the fee for every line is the same. Moreover, we assume the fee to be linear in the distance, i.e., t_s is a linear function of the distance. Linear fare structures are widely used and well-studied in the public transport literature [81]. For each edge e , the total distance that is covered by travelers is equal to the product of the length of the link, ℓ_a , and the flow x_a . To get the distance covered in the entire network, we sum over all the edges $a \in \mathcal{A}_l^t$ for each line l . Mathematically, the operator revenue can be expressed as follows:

$$t_s \left(\sum_{l \in \mathcal{L}} \sum_{a \in \mathcal{A}_l^t} \ell_a \cdot x_a \right). \quad (9)$$

The operator costs are a function of the number of vehicles required and the total length of every line. The length of line l is defined as the sum of the lengths of all edges of that line:

$$\ell_l = \sum_{a \in \mathcal{A}_l^t} \ell_a. \quad (10)$$

The required fleet size per hour for that line can be expressed as

$$\sigma_l = \frac{2 \cdot \ell_l}{v_l} \cdot f_l, \quad (11)$$

where v_l was defined as the speed of the line and f_l as the frequency. Here, we assume that a line operates both ways. Consequently, the operator costs can be defined as

$$O \cdot T_o \cdot \sum_{l \in \mathcal{L}} \sigma_l \ell_l, \quad (12)$$

where O is the operating cost per kilometer for one vehicle and T_o is the number of operating hours considered. For now, we also assume O to be a constant. However, by allowing for line-specific fees and operating costs in the same way as we do for the frequencies and speeds, we could eliminate this assumption. Note that the flows, generalized costs, and demand for both car and transit users are implicitly a function of the decision variables, i.e., line routes, line frequencies, and parking fees. In conclusion, the objective functions can be summarized as

$$\begin{aligned} Z(\mathcal{L}, \vec{f}, \vec{p}) = & \left(\sum_{(o,d) \in S} \left(c_{\text{car}}^{o,d} \cdot q_{\text{car}}^{o,d} + \sum_{a,e \in \{\text{walk}, \text{cycle}\}} c_{\text{transit},a,e}^{o,d} \cdot q_{\text{transit},a,e}^{o,d} \right) \right. \\ & \left. + \sum_{(o,d) \in S} q_{\text{car}}^{o,d} \right. \\ & \left. + O \cdot T_o \cdot \sum_{l \in \mathcal{L}} \sigma_l \ell_l - t_s \left(\sum_{a \in \mathcal{A}^t} \sum_{l: a \in \mathcal{A}_l^t} \ell_a x_a \right) \right). \end{aligned} \quad (13)$$

3.5 | Optimization Problem

We now formally define the TNDFPSP as a multiobjective optimization problem. The goal is to determine the set of transit lines $\mathcal{L}$, their frequencies $\vec{f} = (f^l)_{l \in \mathcal{L}}$, and parking fees $\vec{p} = (p^i)_{i \in \mathcal{N}_p}$ that aim to simultaneously minimize the three objectives defined previously. We define the vector of frequencies as $\vec{f} = (f^l)_{l \in \mathcal{L}}$ and the vector of parking fees as $\vec{p} = (p^i)_{i \in \mathcal{N}_p}$. We also define the matrices of car generalized costs, demand, and flows as $\mathbf{c}_{\text{car}} = (c_{\text{car}}^{o,d})_{(o,d) \in S}$, $\mathbf{q}_{\text{car}} = (q_{\text{car}}^{o,d})_{(o,d) \in S}$, and $\mathbf{x}_{\text{car}} = (x_a)_{a \in \mathcal{A}^t}$. In the same way, we define $\mathbf{c}_{\text{transit}} = (c_{\text{transit},a,e}^{o,d})_{(o,d) \in S}^{a,e \in \{\text{walk}, \text{cycle}\}}$, $\mathbf{q}_{\text{transit}} = (q_{\text{transit},a,e}^{o,d})_{(o,d) \in S}^{a,e \in \{\text{walk}, \text{cycle}\}}$, and $\mathbf{x}_{\text{transit}} = (x_a)_{a \in \mathcal{A}^t}$. We assume the number of lines is some fixed number, $\#L$. As argued by Moghaddam et al. [82], a low number of stops increases the required transfers, while a high number of stops increases costs per kilometer. Thus, we impose a minimum and maximum number of nodes for each line, $\mathcal{N}_{\min}^t$ and $\mathcal{N}_{\max}^t$, respectively. Lastly, for both frequencies and parking fees, we introduce a set of possible values, which we denote by $\mathcal{F}$ and $\mathcal{P}$, respectively. In conclusion, the bi-level, multiobjective optimization problem is given by

$$\min_{\mathcal{L}, \vec{f}, \vec{p}} Z(\mathcal{L}, \vec{f}, \vec{p}), \quad (14)$$

$$\text{s.t. } p^i \in \mathcal{P}, \quad \forall i \in \mathcal{N}_p, \quad (15)$$

$$|\mathcal{L}| = \#L, \quad (16)$$

$$\mathcal{N}_{\min}^t \leq \mathcal{N}_l^t \leq \mathcal{N}_{\max}^t, \quad \forall l \in \mathcal{L}, \quad (17)$$

$$\mathcal{A}_l^t \text{ is connected and acyclic in } \mathcal{G}^t, \quad \forall l \in \mathcal{L}, \quad (18)$$

$$f^l \in \mathcal{F}, \quad \forall l \in \mathcal{L}, \quad (19)$$

$$(\mathbf{c}_{\text{car}}, \mathbf{q}_{\text{car}}, \mathbf{x}_{\text{car}}) \in \text{UE}, \quad (20)$$

$$(\mathbf{c}_{\text{transit}}, \mathbf{q}_{\text{transit}}, \mathbf{x}_{\text{transit}}) \in \text{TS}, \quad (21)$$

$$c_{\text{transit}}^{o,d} = -\tau \cdot \log \left(\sum_{a,e \in \{\text{walk}, \text{cycle}\}} \exp \left(-\frac{1}{\tau} \cdot c_{\text{transit},a,e}^{o,d} \right) \right), \quad \forall (o, d) \in S, \quad (22)$$

$$q_m^{o,d} = q^{o,d} \cdot \frac{\exp(-\theta c_m^{o,d})}{\exp(-\theta c_{\text{car}}^{o,d}) + \exp(-\theta c_{\text{transit}}^{o,d})}, \quad \forall (o, d) \in S, m \in M, \quad (23)$$

where $M = \{\text{car}, \text{transit}\}$ are the two main modes we consider. Equation (14) addresses the objective function we aim to minimize. Because of the mentioned conflicting objectives, our aim is not to find one unique solution but to find the Pareto front. Constraints (15)–(19) are the constraints on the decision variables.

Constraints (20)–(23) reveal the bilevel nature of the optimization problem. In particular, Constraint (20) states that the generalized costs, the demand, and flows for car traffic correspond to the UE as defined in Section 3.3.1. Constraint (21) states that the generalized costs, demand, and flows for public transport correspond to the Zenith algorithm as introduced in Section 3.3.2, which we indicate by Transit Solution (TS). Constraints (22) and (23) reflect the modal split discussed in Section 3.3.3.

3.6 | Summary of Key Modeling Assumptions

The main modeling assumptions made in Section 3 are summarized below. Several of these (e.g., constant parking duration and constant transit fares) are adopted for clarity and simplicity but can be relaxed within our framework if more detailed modeling is required.

- The car and transit networks are treated separately; integration occurs via mode choice and aggregate flows.
- Every OD pair is connected in the car network.
- All nodes can serve as zonal centroids, road intersections, and transit stops.
- Car access time is zero; egress walk time can be nonzero based on the parking location choice.
- Park search time is modeled via dedicated parking links at each intersection; average parking duration is assumed constant.
- Walking and cycling are allowed as access and egress modes for transit, but only walking is allowed for transfers.
- Delay functions for transit, walking, and cycling links are constant, depending on the length and speeds.
- Fare functions are the same for each line and are linear in distance; operating costs per kilometer are also the same for each line.

- Road congestion is modeled for cars using BPR-type delay functions; car travel times increase with flow.
- No explicit vehicle capacity constraints or in-vehicle crowding effects are modeled for public transport; in-vehicle transit travel time is not affected by car congestion.

4 | Solution Method

Finding exact solutions for the TNDFPSP is computationally infeasible for reasonably sized networks due to its NP-hardness. We therefore approximate the Pareto front using metaheuristics. Among these, NSGA-II is particularly well-suited for multi-objective problems with discrete decision variables and three objectives, as in our case, and is widely used in TNDFSP literature for its balance between convergence, diversity, and straightforward implementation.

Figure 2 presents the flowchart of our extended NSGA-II algorithm. Each solution encodes transit routes, frequencies, and parking fees. Solutions are evaluated using the multimodal assignment model (Section 3.3) and ranked by nondominance. Among solutions with the same nondominance rank, the crowding distance is used to promote diversity. Parent selection, crossover, and mutation are based on existing NSGA-II applications to solve the TNDFSP but are adapted to handle parking fees. We also introduce a repair operator for connectivity and an expansion operator, applied after the final iteration, to further extend the Pareto front.

The following subsections describe the algorithmic components specific to our application: the representation of solutions and initialization procedure (Section 4.1), the adaptation of variation operators to incorporate parking fees (Section 4.2), and the additional repair and expansion operators (Section 4.3). Other elements of the NSGA-II procedure follow established practice and are described in detail by Deb et al. [59].

4.1 | Solution Representation and Initialization

Each solution is represented by a set of transit lines, each defined by a sequence of nodes (the route) and an associated frequency, along with a parking fee for each parking node. Formally, a solution can be represented as follows:

$$\left(\left(\mathcal{N}_{l_1}^t, f^{l_1} \right), \dots, \left(\mathcal{N}_{l_{|L|}}^t, f^{l_{|L|}} \right), p^1, \dots, p^{|\mathcal{N}_P|} \right). \quad (24)$$

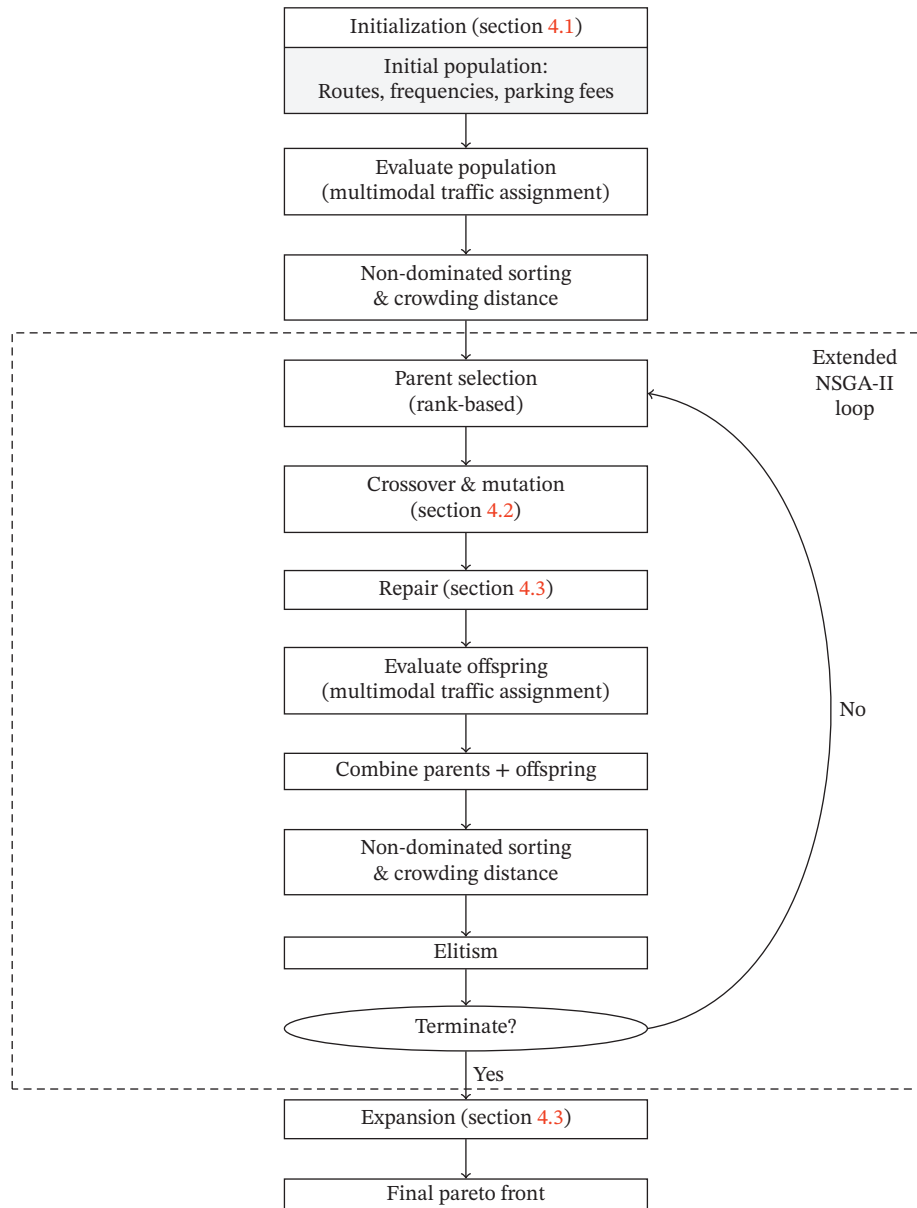


FIGURE 2 | Flowchart of the extended NSGA-II: the repair step may not succeed in the multimodal setting; the expansion operator is applied once after the final iteration.

A visualization of such a representation is shown in Figure 3. For example,

$$((4, \dots, 5, 15), \dots, (1, \dots, 6, 4), 3, \dots, 2), \quad (25)$$

indicates that line 1 traverses nodes 4, 3, 2, 5 with a frequency of 15 vehicles per hour and that the parking fee at the first parking node is 3. Note that the number of nodes in each line (i.e., the route length) is dynamic and may vary between lines but is always constrained by the minimum and maximum allowed values (see Constraint (17)).

To generate initial solutions, we create $\#L$ routes with corresponding frequencies and assign a parking fee to each parking node. Following Moghaddam et al. [82], we generate a pool of feasible routes using Yen's k -shortest path algorithm [83], then apply a simple heuristic to reduce the pool size, balancing solution quality and computational efficiency. In particular, routes

within the node constraints (Constraint (17)) and with a length at most 1.5 times the shortest path are considered.

For each solution, $\#L$ routes are selected from the pool with probabilities proportional to the direct service measure proposed by Nikolić and Teodorović [84], defined as the demand directly satisfied by the shortest path between an OD pair:

$$ds^{o,d} = \sum_{i,j \in \mathcal{N}_{sp}^{o,d}} q^{ij}. \quad (26)$$

Moreover, although not necessary, having every OD pair served by transit is desirable. To promote network coverage, we ensure that each new route includes at least one node not yet present in the selected routes, until all nodes are covered or until the maximum number of routes is reached.

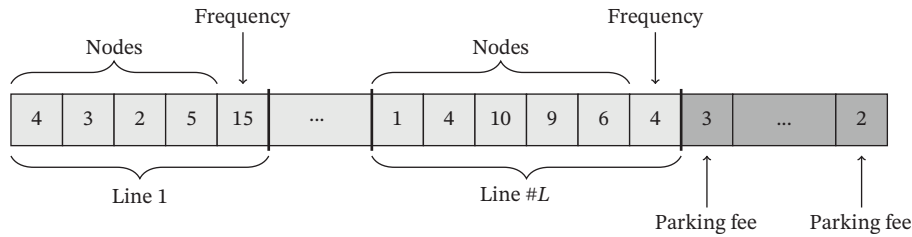


FIGURE 3 | An example of the representation of a solution.

Unlike most studies, which assume fixed transit demand and set frequencies to accommodate all passengers [35, 36, 55, 85, 86], our approach considers variable transit demand in a multimodal network. Since frequencies affect transit travel time and thus generalized costs, there is feedback between service provision and demand. Therefore, frequencies and parking fees are treated as explicit decision variables, and initial values are generated uniformly at random from their respective feasible sets $\mathcal{F}$ and $\mathcal{P}$, allowing NSGA-II to optimize them.

4.2 | Crossover and Mutation

We now describe the crossover and mutation operators used in our evolutionary algorithm. In each iteration, parent solutions are selected using rank-based selection, and crossover and mutation are applied to generate offspring. We extend the operators introduced by Durán-Micco et al. [36] to handle parking fees as decision variables.

The crossover operator, based on the one proposed by Mumford [60], creates offspring by alternately selecting half of the routes from each parent. Routes are chosen with probability proportional to the number of new nodes they introduce. To encourage connectivity and transfer possibilities, only lines sharing at least one node with existing routes are considered; otherwise, selection is uniform at random. Each line's frequency is inherited from the parent providing the route, and route lengths are constrained as in Constraint (17). To incorporate parking fees, we apply a single-point crossover: a random cutoff among the parking fees is selected, and the fees after this point are swapped between parents. Usually, crossover is applied to an individual with high probability, which we call the crossover probability. To facilitate convergence, we set the crossover probability equal to one.

After crossover, mutation is applied with a small probability. If mutation occurs, we use the operator presented by Nayeem et al. [87] for transit routes, which acts on single routes: with probability 0.7, a small modification is made, either deleting a terminal node (with probability 0.4) or adding a neighbor of a terminal node; otherwise, a large modification is performed, where a new terminal node is chosen and the shortest path between this node and an existing terminal node is added to the line. Mutations are only accepted if the resulting route length satisfies Constraint (17). For frequencies and parking fees, mutation is performed by uniformly sampling new values from $\mathcal{F}$ and $\mathcal{P}$, respectively. Based on the literature and initial tests, we set the mutation probability to 0.1, similar to values used in existing work [34, 36, 55, 61].

4.3 | Repair and Expansion

In addition to the classical operators of the NSGA-II framework, we consider two additional operators to improve the solution quality: a repair operator and an expansion operator.

The repair operator, used in several TNDP studies, is applied after each crossover and mutation step to promote network connectivity. If a solution's routes do not serve all OD pairs, neighboring nodes are iteratively added to route terminals until all OD pairs are served or no further additions are possible. Unlike traditional repair strategies, our multimodal setting allows for feasible solutions that do not serve all OD pairs; in such cases, unsatisfied demand is entirely assigned to car travel, which may negatively affect objectives.

We propose an expansion operator that is applied after the final iteration to increase the size of the Pareto front by exploiting relationships between decision variables and objectives. Unlike local search procedures, which are computationally demanding in multimodal settings, our expansion operator is only applied after convergence.

To maximize modal shift, we make transit as attractive as possible by choosing maximal frequencies, i.e., $f^l = \max \mathcal{F}$ for all lines l and by making cars as unattractive as possible by choosing maximal parking fees, i.e., $p^i = \max \mathcal{P}$ for all parking nodes i . Moreover, it makes sense to maximize the length of the lines within the constraint on the number of nodes given in Constraint (17) to cover as much demand as possible. To achieve this, we iteratively add nodes until the maximum number of nodes is reached or if no further nodes can be added. We examine all neighboring nodes of both terminal nodes and select the one that maximizes the total demand covered. We apply this to the ten solutions with the smallest number of car users and thus the largest modal shift towards transit. We then check if they are part of the Pareto front and otherwise discard these new solutions. We have also considered applying this to five and fifteen solutions with the smallest number of car users, respectively, but ten seemed the most suitable. In the case of five solutions, some new solutions remained undiscovered, while with fifteen solutions, the additional new solutions did not provide fewer car users.

On the other hand, to minimize generalized costs, we make transit as unattractive as possible by choosing minimal frequencies, i.e., $f^l = \min \mathcal{F}$ for all lines l and by making cars as attractive as possible by choosing minimal parking fees, i.e., $p^i = \min \mathcal{P}$ for all nodes i . We apply this to the ten solutions with the lowest generalized costs. Again, we check if they are part of the Pareto front and discard these new solutions if not. Removing

nodes from routes could further reduce user costs, but would result in disconnecting the network, so we do not include this in the operator.

We have also considered applying this expansion operator to the population during earlier iterations. However, this negatively affected the quality of the solutions in terms of all three objectives and did not give any improvements compared to applying the expansion operator in the last iteration. This can be attributed to the nature of the NSGA-II framework.

While the expansion operator can help find solutions optimized for a single objective, the goal of multiobjective optimization is to explore trade-offs. Nevertheless, the expansion operator provides a computationally efficient way to increase the size of the Pareto front to get more insights into the trade-off between objectives.

5 | Experimental Design

To evaluate our algorithm and assess the benefits of optimizing parking fees within the TNDSP, we conduct a series of computational experiments. This section describes the experimental setup, beginning with the network, followed by the parameter values selected throughout the experiments.

5.1 | Multimodal Version of Mandl's Network

We apply our solution method (Section 4) to Mandl's network [88], the most widely used benchmark for TNDSP studies [11]. Mandl's network consists of 15 nodes and 21 links, representing a group of cities in Switzerland, and serves as a standard test case for methodological comparison.

For every link, the travel time by bus is provided. As we also consider trips made by car, we convert the transit travel times to distances using the speed equation $v = s/t \implies s = v \cdot t$, where v is the speed, s the distance, and t the travel time. Consistent with Durán et al. [35], we assume a bus speed of 45 km/h and a car speed of 60 km/h. The resulting network, with distances in kilometers on every link, is visualized in Figure 4.

Mandl's network is symmetric: links go in both directions. The demand for public transport for every OD pair during peak time is also provided. The (transit) OD demand is again symmetric and is given in the OD matrix shown by Table 1. We assume that the total demand is simply a multiple of the transit demand, as there is no data regarding the total demand. Therefore, we take the total demand equal to the transit demand and choose road and parking capacities accordingly, which we will define later.

Because Mandl's network provides information only at the city level, we assume all nodes serve as zonal centroids, road intersections, and transit stops. For the car subnetwork, road links coincide with those in Mandl's network, and each intersection has a corresponding parking node and link to model park search time. We assume car drivers can reach their destination directly from any parking node, so egress costs are zero.

As discussed in Section 3.1, we take $\alpha_a = 2$ and $\beta_a = 4$ for the delay functions on road links $a \in \mathcal{A}_r$ in Equation (2). Free-flow travel time can be deduced from the car speed and the length of every link, as given in Figure 4.

We choose a capacity of 15,000 vehicles for each link, which roughly equals the total travel demand for the entire network.

This choice ensures that, due to the nonlinear BPR delay function, travel times still increase substantially as flows grow. Consequently, the network exhibits moderate congestion effects while avoiding severe congestion. Our primary aim is to analyze the interaction between parking policies, transit network design, and modal choice under these conditions. The parameter values are summarized in Table 2.

As congestion levels may influence the effectiveness of parking pricing policies, we additionally consider a substantially more congested setting in the sensitivity analysis of Section 6.4, where road capacities are reduced, allowing us to assess the robustness of the main conclusions with respect to the assumed congestion level.

Transit links do not necessarily have to cover the entire network, but transit links are only allowed between nodes that are connected in Mandl's network. As argued in Section 3.1, we take in-vehicle transit travel time to be constant and equal to the free flow travel time, which can be deduced from the bus speed and the length of every edge. Like the study of Kim et al. [33], we consider access and egress distances to be 200 m for every transit user. We assume bicycle speed to be 15 km per hour and walking speed to be 4 km per hour. This means access or egress by bicycle takes 1/75 h, while access or egress by walking takes 1/20 h.

All assumptions and simplifications above are specific to our multimodal version of Mandl's network for experimental purposes and do not restrict the general applicability of our modeling framework.

5.2 | Parameter Values

Now that the multimodal version of Mandl's network is defined, we need to specify a set of parameters in light of the generalized costs, multimodal assignment model, optimization problem, and the NSGA-II framework.

We start by specifying the parameter values concerning the car and transit costs as given in Equations (3) and (4), respectively. As argued by Shen et al. [89], generalized costs for transit passengers can increase when, for example, there is no seating available. To add this potential extra discomfort, we have chosen the mode-specific constant for transit equal to two for all access and egress modes. Formally, this means that $\mu_{a,e} = 2$ for all $a, e \in \{\text{walk, cycle}\}$. As the car does not bring such discomfort, the mode-specific constant for cars, μ_{car} , is chosen as 0. We continue with the VoT parameters. Mandl [88] does not use VoT parameters, and therefore, no values are provided. Wardman [90] provides an analysis of VoT values across Europe, including Switzerland. More recently, Kouwenhoven et al. [91] conducted such research only for the Netherlands. Based on these two studies, we have chosen all the VoT parameters equal to ten euros per hour as an approximation of the real values. More accurate values could be found by analyzing VoT values for a specific region, in this case, the group of cities that Mandl's network represents. However, this falls outside the scope of this study. Because of the uncertainty in the mode-specific constants and the VoT values, we provide a sensitivity analysis in Section 6.4 to analyze the robustness of our model to these uncertainties.

In addition to time-related parameters, we specify (monetary) cost-related parameters. Data on the average parking duration in Switzerland is limited: Maurer et al. [92] reports roughly 45 min

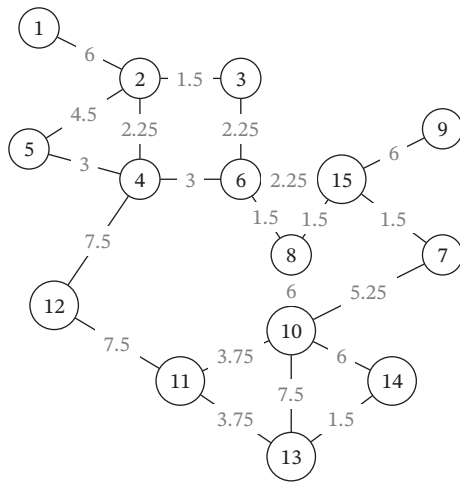


FIGURE 4 | Mandl's network with distances in kilometers.

in a small part of Zurich, but with minimal data. More extensive research in the Netherlands by Mingardo et al. [93] finds an average duration of about 2 hours, which we adopt in our model. Since parking duration directly scales the monetary impact of parking fees and therefore strongly influences the effectiveness of parking-pricing policies, we additionally investigate the sensitivity of the results to this parameter in Section 6.4. For the fuel price per kilometer, c_s , we use a value of 0.1 euros per kilometer, based on recent fuel prices [94].

For the public transport fee, we have selected the function $t_s(d) = 0.78 + 0.22d$, which is based on current prices in practice. The penalty for making a transfer is set to 5 min, consistent with the majority of literature on the TNDFSP concerning Mandl's network (e.g., [35, 36, 60, 86, 87]). To map this to a monetary value, we first convert this to 5/60 h and then multiply by a VoT value of ten to obtain a penalty of 5/6 for each transfer. All parameter values regarding the generalized costs for cars and transit are displayed in Table 3.

Concerning the car assignment model, we have selected the number of iterations for the Frank–Wolfe algorithm, $nc_{\max}$, as ten. This value was found through initial experiments showing that the improvements in the generalized costs on an OD level are minimal after the first ten iterations. This number is relatively small, as no major congestion occurs. On the other hand, some iterations are still required as travel times significantly change with different flows because of the chosen BPR functions. For the nested logit model, we have normalized the logit scale parameter θ for the choice between car and transit in Equation (6) to one. As we do not have information about access and egress, we choose the dissimilarity parameter τ in Equation (5) to be small, i.e., 0.05. This way, users still perceive a difference when choosing another access or egress mode for public transport, but this difference is relatively small. Furthermore, as proposed in [95], we choose the line choice parameter τ_{line} equal to two. The parameter values used for the multimodal assignment model are given in Table 4.

We proceed to the parameter values regarding the objectives and constraints in the optimization problem. In the objective function, we defined in Equation (13), we introduced the variable O , which indicates the operating costs of one bus for 1 km, and the variable T_o specifies the number of operating hours we consider. It is hard to find an approximate value, as it consists of many components that are difficult to express as costs per kilometer [96]. We have chosen twenty euros per kilometer to balance the effect of the fleet size and fare revenues on the operator deficit. Given the uncertainty in this value, we also include it in the sensitivity analysis. Moreover, as we consider the peak time, we assume $T_o = 1$. It should be noted that a sensitivity analysis of O also assesses the sensitivity of our model to changes of T_o . Hence, we do not conduct a separate sensitivity analysis of T_o . In our optimization problem, we defined constraints (15)–(19) on the decision variables, which also involve parameters. In [91], the average parking fee in their study of the Netherlands is 1.93 euros an hour. Therefore, as the possible parking fees for each city, denoted by $\mathcal{P}$, we choose a range from 0.25 to 2.5 euros per hour with steps of 0.25 euros. Furthermore, the possible frequencies,

TABLE 1 | Mandl's network: OD matrix.

O\D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	400	200	60	80	150	75	75	30	160	30	25	35	0	0
2	400	0	50	120	20	180	90	90	15	130	20	10	10	5	0
3	200	50	0	40	60	180	90	90	15	45	20	10	10	5	0
4	60	120	40	0	50	100	50	50	15	240	40	25	10	5	0
5	80	20	60	50	0	50	25	25	10	120	20	15	5	0	0
6	150	180	180	100	50	0	100	100	30	880	60	15	15	10	0
7	75	90	90	50	25	100	0	50	15	440	35	10	10	5	0
8	75	90	90	50	25	100	50	0	15	440	35	10	10	5	0
9	30	15	15	15	10	30	15	15	0	140	20	5	0	0	0
10	160	130	45	240	120	880	440	440	140	0	600	250	500	200	0
11	30	20	20	40	20	60	35	35	20	600	0	75	95	15	0
12	25	10	10	25	15	15	10	10	5	250	75	0	70	0	0
13	35	10	10	10	5	15	10	10	0	500	95	70	0	45	0
14	0	5	5	5	0	10	5	5	0	200	15	0	45	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 2 | Parameter values for road and parking links.

	α_a	β_a	c_a
Road	2	4	15,000
Parking	4	9	15,000

denoted by $\mathcal{F}$, range from two to twenty vehicles per hour, which is reasonable considering current frequencies around Europe. Consistent with studies on the TNDP using Mandl's network, the minimum number of nodes $\mathcal{N}_{\min}^t$ for a line is two and the maximum $\mathcal{N}_{\max}^t$ is eight [36, 56, 60, 62, 84, 97]. We look at four, six, seven, and eight lines, following the previously mentioned existing research. All these parameter values are displayed in Table 5.

Lastly, we discuss the parameter values used for the NSGA-II framework. We choose the crossover, mutation, small modification, and deletion probabilities as discussed in Section 4.2. We aimed to find appropriate values for the population size and maximum number of iterations by running preliminary experiments. In these experiments, it turned out that in order to obtain a diverse set of solutions with low objective values, considering a large population size is more important than a high maximum number of iterations. As a result, we have chosen a population size of 500 and a maximum number of iterations of 100. The choices regarding the parameters for the NSGA-II framework are shown in Table 6.

6 | Results

This section presents the results of applying our algorithm to Mandl's network, as described in the previous section. We begin by benchmarking our approach against state-of-the-art algorithms for the classical TNDP. Next, we demonstrate the benefits of extending the problem to include frequencies and, most importantly, parking fees as decision variables, by comparing the TNDPSP to the classical TNDP. We demonstrate that objective values can be reduced significantly and that our solution method allows a broad trade-off analysis. We then assess the improvement of the Pareto fronts by applying our proposed expansion operator. Furthermore, we conduct a sensitivity analysis on key parameters to assess the robustness of our model. Finally, we elaborate on computational considerations and the scalability of our model.

6.1 | Comparison With State of the Art

To evaluate the performance of our algorithm, we first conduct experiments on Mandl's network and compare our results with those reported in six state-of-the-art TNDP studies: Mumford [60], Nikolić and Teodorović [84], Chew et al. [56], Kechagiopoulos and Belgiannis [62], Buba and Lee [97], and Durán-Micco et al. [36]. These studies solve the TNDP for Mandl's network with 4, 6, 7, and 8 lines, where each line consists of at least two and at most eight nodes. All of them consider a passenger-based objective, such as minimizing average transit travel time, while Mumford [60] and Chew et al. [56] also include minimizing operator costs as an objective. In these cases, we include both the passenger-oriented and operator-oriented solutions.

None of these studies includes parking fees or a bimodal setting. Furthermore, some studies do not include frequencies et al., while those that do use very different ranges. For a fair comparison, we fix parking fees and frequencies at the midpoints of their ranges (i.e., 1.25 and 10, respectively), so only line routes are decision variables.

Because the original studies use different assumptions and models, a direct comparison of their reported objective values would be inconsistent. Therefore, we re-evaluate all literature solutions using our multimodal traffic assignment model (Section 3.3) to compute the three objectives: generalized user costs, number of car users, and operator deficit. Although these literature solutions were obtained for simpler TNDP and TNDPSP formulations, they represent established benchmark solutions. Re-evaluating them within our multimodal framework provides a consistent baseline for assessing the value of the additional decision variables considered in the TNDPSP.

Our algorithm was run 16 times per scenario due to its stochastic nature and the architecture of the computing cluster we used. It is difficult to compare the quality of Pareto fronts, especially when more than two dimensions are considered [98]. As the existing literature that we consider either aims to minimize the number of car users (by minimizing transit travel times) or operator costs, we selected fronts based on these two objectives. More precisely, we selected the Pareto front with the best combined rank among the sixteen fronts on car users and operator deficit.

From the front, we report three representative solutions: UC-MIN, CU-MIN, and OD-MIN, minimizing generalized user costs, car users, and operator deficit in Table 7, respectively. Each block corresponds to a fixed number of lines. For every solution, we show its objective values and, in the last column, #PF dom., the number of solutions of the Pareto-front obtained by our algorithm that dominate it. For completeness, the corresponding line configurations for UC-MIN, CU-MIN, and OD-MIN are provided in Appendix B, Table A1.

Two main observations appear:

1. **Single-objective performance:** For all scenarios, the minimizers from our algorithm outperform the corresponding literature solutions on their targeted objective. UC-MIN always achieves the lowest user costs, CU-MIN the lowest car users, and OD-MIN the lowest operator deficit, even when compared to operator-oriented designs such as M13-O and CLS13-O.
2. **Pareto dominance:** Almost all literature solutions are dominated by solutions from the Pareto front obtained by our algorithm, except for the operator-oriented ones at 7 and 8 lines. This could be explained by the cost-benefit dynamics: adding lines has a large impact on costs, while the corresponding increase in fare revenue becomes smaller beyond a certain number of lines. However, even in these cases, OD-MIN achieves a lower operator deficit.

The complete Pareto fronts are visualized in Figure 5. The x-axis indicates the generalized user costs, the y-axis denotes the number of car users, and the color denotes the operator deficit. The objective values of the solutions obtained in the existing literature are also plotted and marked by a diamond shape. On

TABLE 3 | Parameter values for generalized costs.

Mode	μ_{car}	VoT_{car}	$\text{VoT}_{\text{search}}$	VoT_{walk}	T_{park}	c_{s}	$\mu_{a,e}$	$\text{VoT}_{\text{cycle}}$	$\text{VoT}_{\text{transit}}$	VoT_{wait}	$t_{\text{s}}(d)$	λ
Car	0	10	10	10	2	0.1						
Public Transport				10			2	10	10	10	$0.78 + 0.22d$	5/6

TABLE 4 | Parameter values for the multimodal assignment.

Parameter	Value
n_{Cmax}	10
θ	1
τ	0.05
τ_{line}	2

TABLE 5 | Parameter values for the optimization problem.

Parameter	Value
O	20
T_o	1
$\mathcal{P}$	$\{0.25, 0.5, \dots, 2.5\}$
$\mathcal{F}$	$\{2, 3, \dots, 20\}$
$\mathcal{N}_{\min}^t$	2
$\mathcal{N}_{\max}^t$	8
$\#L$	4, 6, 7, 8

the operator side, the solutions found by Mumford [60] and Chew et al. [56] are labeled in each plot. However, other solutions all have similar objective values and are located in the dotted squares. The zoomed-in versions of these squares are provided in Appendix A in Figure A1.

Overall, our algorithm not only achieves better objective values for each single-objective criterion but also provides a diverse set of Pareto-efficient solutions, enabling explicit trade-off analysis across user costs, car users, and operator deficit, something not addressed in existing studies.

6.2 | Impact of Considering the Optimization of Frequencies and Parking Fees

Now that our algorithm demonstrates strong performance on the TNDP with constant frequencies and parking fees, we evaluate the impact of treating frequencies and, more importantly, parking fees as decision variables. This allows us to distinguish between improvements obtained from optimizing transit routes alone, improvements obtained from jointly optimizing routes and frequencies, and improvements obtained from combining these transit-related decisions with parking policies.

As described in Section 5.2, we consider frequencies in the set $\mathcal{F} = \{2, \dots, 20\}$ and parking fees in the set $\mathcal{P} = \{0.25, \dots, 2.5\}$ for optimization. We therefore consider three problem variants: (i) the classical TNDP, where only routes are optimized, (ii) the TNDPSP, where routes and frequencies are optimized, and (iii) the full TNDPSPSP, where routes, frequencies, and parking fees are optimized. The resulting solutions are compared in terms of objective values, Pareto-front quality, and their implications for policy making.

6.2.1 | Solution Quality and Pareto-Front Coverage

For each scenario and number of lines, we run the algorithm 16 times, select the best Pareto front as before, and report representative solutions in Table 8. Solutions minimizing user costs, car users, and operator deficit are again denoted as UC-MIN, CU-MIN, and OD-MIN, with suffixes-F (frequencies optimized) and -FP (frequencies and parking fees optimized). Figure 6 visualizes the resulting Pareto fronts, where a z-axis for the operator deficit is added instead of the use of a color bar. We have applied a quadratic approximation to create a surface from the obtained solutions by minimizing the squared sum of residuals. The corresponding line configurations, frequencies, and parking fees for the reported solutions are provided in Appendix B, Table A2.

The results show that optimizing frequencies improves all objectives and increases solution diversity, enabling a better exploration of trade-offs. Incorporating parking fee optimization further reduces minimal user costs and car usage across all scenarios. While the effect on operator deficit is less pronounced, since it is largely driven by fleet size and line length, optimizing parking fees generally yields solutions with lower deficits.

To complement the visual comparison of the Pareto fronts in Figure 6, we quantify the quality of the obtained fronts using two multiobjective performance indicators: the hypervolume (HV) indicator [99] and the Inverted Generational Distance (IGD) [100]. HV measures the volume of objective space dominated by a Pareto front relative to a common reference point, where larger values indicate better spread and convergence. The IGD measures the average distance between a Pareto front and a reference Pareto front, where smaller values indicate a closer approximation.

Since the true Pareto front is unknown, we construct an empirical reference front by taking the nondominated union of the solutions obtained for the three problem variants (without parking and frequencies, without parking but with frequencies, and with both parking fees and frequencies). Before calculating the indicators, all objectives are normalized using the minimum and maximum objective values observed across this union. HV is computed using a common reference point of (1.1, 1.1, 1.1) in the normalized objective space. Since all objectives are normalized to the interval [0, 1], this reference point lies slightly beyond the normalized nadir point and is therefore dominated by all considered Pareto fronts. IGD is evaluated with respect to the empirical reference front. The resulting indicator values are reported in Table 9.

The obtained values quantitatively confirm the observations from Figure 6. Across all four scenarios, the variant including both parking-fee and frequency optimization achieves the highest HV and the lowest IGD. The two obtained fronts without parking fees optimization attain only approximately 35%–39% of the HV of the front of the full TNDPSPSP.

TABLE 6 | Parameter values for the evolutionary algorithm.

Parameter	Value
p_c	1
p_m	0.1
p_{sm}	0.7
p_d	0.4
pop_size	500
max_iter	100

Although Table 8 shows that optimizing frequencies can improve the extreme values of individual objectives, the corresponding Pareto fronts remain relatively close to those obtained for the TNDP. Consequently, only limited improvements in HV and IGD are observed. In contrast, incorporating parking-fee optimization substantially increases the range of attainable trade-offs and yields solutions that are consistently closer to the best trade-offs observed in the experiments.

Overall, the HV and IGD indicators confirm that the primary gains arise from jointly optimizing transit and parking policies, while frequency optimization alone provides only limited improvements in the overall quality and coverage of the obtained Pareto fronts.

6.2.2 | Policy Implications

Beyond the objective values themselves, the obtained solutions also have broader implications for sustainability and policy making. As an additional illustration of the implications of these solutions, we estimate the associated CO₂ emissions for the CU-MIN-(FP) solutions with four lines. The resulting calculation should be interpreted as an illustrative estimate based on passenger-kilometers rather than as a detailed emissions assessment. In particular, we estimate total CO₂ emissions using standard modal carbon intensities. For car travel, we use a lower-bound emissions factor of 160 g CO₂ per passenger-kilometer, reflecting the small but growing share of electric vehicles in the car fleet [101, 102]. For bus travel, we use 97 g CO₂ per passenger-

TABLE 7 | Objective values for representative solutions of our algorithm and literature solutions on Mandl's network (constant frequencies and parking fees).

#lines	Solution*	User costs	Car users	Operator deficit	#PF dom.
4	M13-P	57,916	14,851	26,554	8
	M13-O	57,330	15,094	4619	58
	NT13	57,916	14,847	25,695	5
	CLS13-P	57,948	14,832	27,718	7
	CLS13-O	57,330	15,094	4619	58
	KB14	57,932	14,835	27,064	7
	BL18	57,867	14,879	27,810	14
	DVV20	57,961	14,821	23,901	3
	UC-MIN	57,024	15,156	2819	—
	CU-MIN	57,982	14,795	21,597	—
	OD-MIN	57,291	15,097	2009	—
6	M13-P	58,085	14,755	40,256	6
	M13-O	57,383	15,073	3996	63
	NT13	58,073	14,761	41,053	5
	CLS13-P	58,074	14,764	42,155	6
	CLS13-O	57,383	15,073	3996	63
	KB14	58,064	14,766	33,120	1
	BL18	58,063	14,766	38,100	5
	DVV20	58,052	14,771	37,651	5
	UC-MIN	57,011	15,185	2993	—
	CU-MIN	58,075	14,739	32,289	—
	OD-MIN	57,293	15,094	2363	—
7	M13-P	58,120	14,731	50,187	2
	M13-O	57,331	15,092	4173	0
	NT13	58,124	14,728	43,519	1
	CLS13-P	58,108	14,737	41,088	3
	CLS13-O	57,373	15,076	3884	0
	KB14	58,098	14,744	38,216	3
	BL18	58,090	14,757	50,245	4
	DVV20	58,089	14,748	39,193	3
	UC-MIN	57,032	15,145	4121	—
	CU-MIN	58,117	14,724	35,942	—
	OD-MIN	57,178	15,128	3253	—

(Continues)

TABLE 7 | (Continued)

#lines	Solution*	User costs	Car users	Operator deficit	#PF dom.
8	M13-P	58,116	14,735	53,298	2
	M13-O	57,301	15,103	3163	0
	NT13	58,119	14,732	52,912	2
	CLS13-P	58,096	14,742	40,588	2
	CLS13-O	57,316	15,097	2793	0
	KB14	58,102	14,742	52,921	2
	BL18	58,100	14,749	50,569	3
	DVV20	58,112	14,735	35,474	2
	UC-MIN	57,181	15,082	5974	—
	CU-MIN	58,178	14,692	43,252	—
	OD-MIN	57,228	15,119	2698	—

Note: Bold values indicate the lowest objective value among solutions with the same number of lines (4, 6, 7, or 8).

*M13: Mumford [60], NT13: Nikolić and Teodorović [84], CLS13: Chew et al. [56], KB14: Kechagiopoulos and Belgiannis [62], BL18: Buba and Lee [97], DVV20: Durán-Micco et al. [36].

kilometer [101]. These factors are applied to the total passenger-kilometers for each mode, as computed from the flows and link lengths in our model. The results are presented in Table 10.

For this illustrative example, the TNDFPSP solutions are associated with approximately 3.4% lower estimated CO₂ emissions than the baseline TNDP solutions, while optimizing frequencies alone results in only marginal changes. Similar reductions in car usage are observed for the other scenarios (see Table 8), suggesting comparable effects.

Our results also show that the TNDFPSP framework provides valuable insights into broader trade-offs between policy objectives. Table 11 illustrates this using solutions of the TNDFPSP with similar car users as CU-MIN and CU-MIN-F. For CU-MIN, the TNDFPSP solution with comparable car users and low user cost reduces user costs by about 10% and operator deficit by nearly 88%. When prioritizing operator deficit, the reduction exceeds 93% with a 1% decrease in user cost. For CU-MIN-F, similar results are observed. A solution of the TNDFPSP with a similar number of car users and low user costs decreases user costs by about 12% and operator deficit by approximately 82%. The solution focusing on the operator deficit lowers the deficit by approximately 90% and the user costs by roughly 10%. These results confirm that incorporating parking fee optimization alongside frequencies provides substantial benefits across all objectives and expands the range of trade-offs available to decision-makers.

6.3 | Effectiveness of the Expansion Operator

The expansion operator was introduced in Section 4.3 to improve solution quality by exploiting the problem structure. To find solutions with minimal car users, this operator extends lines and, if applicable, maximizes frequencies and parking fees to discourage car use. On the other hand, to find solutions with minimal user costs, the operator keeps lines unchanged for connectivity and, if applicable, minimizes frequencies and parking fees. Its potential impact increases with the number of decision variables: for constant frequencies and parking fees, only line extensions are applied; for constant parking fees,

frequencies are also adjusted; and for the full TNDFPSP, both frequencies and parking fees are modified.

The expansion operator is applied after optimization, so it does not decrease solution quality, a problem observed in prior experiments when using such adjustments during optimization. While the primary goal remains finding trade-off solutions rather than single-objective minimizers, these extensions help improve the extremes of the objectives, which can give insights into the total range of the objectives.

Table 12 reports the effect of the expansion operator for two objectives: user costs for UC-MIN-F and UC-MIN-FP and car users for CU-MIN, CU-MIN-F, and CU-MIN-FP, isolating the contribution of the proposed expansion operator by comparing the obtained objective values before and after its application. Baseline values are without the operator; improved values are with the operator. The case UC-MIN with constant frequencies and parking fees is omitted because the operator does not apply in this setting.

We observe that the expansion operator effectively extends the Pareto front at its extremes for car users in all problem variants and for user costs in the full TNDFPSP, where parking fees are considered. This provides insights into achievable objective minima without extra computational cost or quality loss, complementing trade-off analysis. We can also see that the operator is especially effective in the full TNDFPSP.

6.4 | Sensitivity Analysis

In Section 5, several parameters were identified for which there is considerable uncertainty in their values or for which alternative assumptions may influence the resulting trade-offs. To assess the robustness of the obtained Pareto fronts, we conduct a sensitivity analysis with respect to five parameters: (1) the road capacity, representing different congestion levels, (2) the mode-specific constant for transit, (3) selected VoT parameters, (4) the average parking duration, and (5) the operator cost per kilometer. As preliminary results indicate that the number of lines does not substantially affect the conclusions, the sensitivity analysis is restricted to the case with four lines for clarity. Table 13

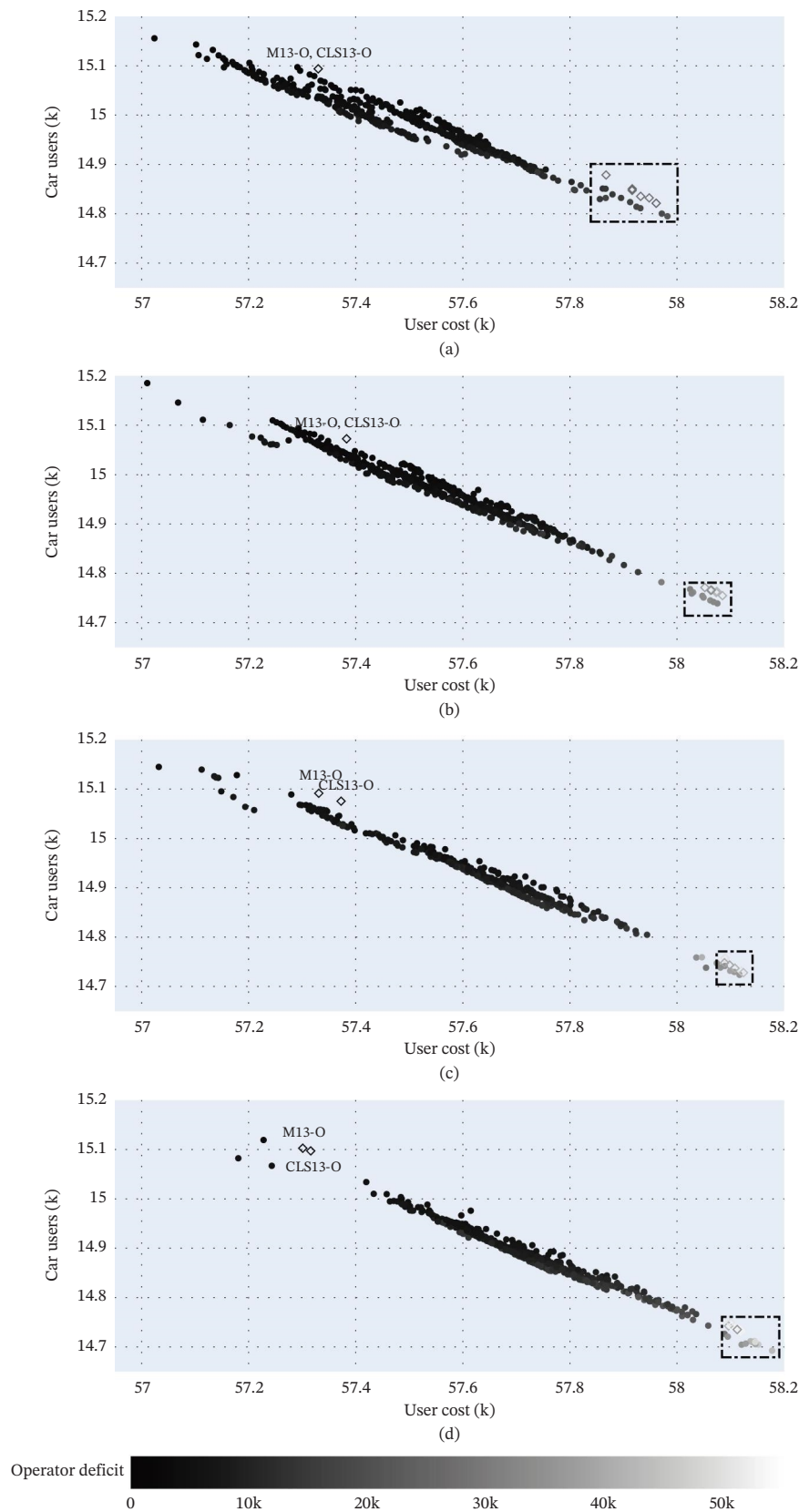


FIGURE 5 | Pareto fronts for constant frequencies and parking fees on Mandl's network. (a) Pareto front for four lines. (b) Pareto front for six lines. (c) Pareto front for seven lines. (d) Pareto front for eight lines.

TABLE 8 | Objective values for representative solutions of our algorithm.

#lines	Solution	User costs	Car users	Operator deficit
4	UC-MIN-F	56,100	15,523	1096
	UC-MIN-FP	40,473	15,544	555
	CU-MIN-F	58,126	14,712	38,051
	CU-MIN-FP	78,856	13,026	45,663
	OD-MIN-F	56,185	15,505	421
	OD-MIN-FP	72,251	14,631	158
6	UC-MIN-F	56,109	15,524	1105
	UC-MIN-FP	40,546	15,528	1182
	CU-MIN-F	58,226	14,658	65,771
	CU-MIN-FP	78,979	12,892	65,537
	OD-MIN-F	56,170	15,505	752
	OD-MIN-FP	68,420	14,554	407
7	UC-MIN-F	56,101	15,526	772
	UC-MIN-FP	40,564	15,521	2241
	CU-MIN-F	58,258	14,644	69,391
	CU-MIN-FP	79,039	12,824	67,327
	OD-MIN-F	56,232	15,493	676
	OD-MIN-FP	66,805	14,818	699
8	UC-MIN-F	56,275	15,481	1804
	UC-MIN-FP	40,550	15,526	1453
	CU-MIN-F	58,292	14,624	82,266
	CU-MIN-FP	79,055	12,795	100,086
	OD-MIN-F	56,396	15,441	904
	OD-MIN-FP	69,200	14,289	827

Note: The *F* indicates frequencies are included as decision variables, *FP* includes both frequencies and parking fees as decision variables. Bold values indicate the lowest objective value among solutions with the same number of lines (4, 6, 7, or 8).

summarizes the parameters considered, their roles in the model, and the motivation for their inclusion.

6.4.1 | Congestion

We first investigate the influence of congestion by reducing the road capacity from $c_a = 15000$ to $c_a = 1500$. The resulting Pareto front is shown in Figure 7a, while the corresponding HV and IGD values are reported in Table 14. The objective minima for the different variants are shown in Table A2 in Appendix C. As expected, increased congestion makes car travel less attractive, resulting in a larger potential for a modal shift towards public transport. Consequently, transit frequency optimization becomes slightly more effective than in the base scenario, as demonstrated by the increase in the relative HV of the TNDFSP compared to the TNDFP. Nevertheless, the TNDFPSP still clearly dominates both alternative variants, obtaining the largest HV and lowest IGD values. The HV of the TNDFSP and TNDFP correspond to only 37.2% and 33.3% of the TNDFPSP HV, respectively. However, parking-fee optimization remains the primary driver of Pareto-front improvement, indicating that its effectiveness is not limited to moderately congested networks.

6.4.2 | Mode-Specific Constant

The next parameter we consider is the mode-specific constant for transit as given in Table 3. As discussed, we add some discomfort to the generalized costs of using transit, consistent with both theory and practice. We have selected a value of two for each

access and egress mode, i.e., $\mu_{a,e} = 2$ for $a, e \in \{\text{walk, cycle}\}$. Now, we analyze the effects of changing this value. As the transit utility as given in Equation (4) is linear in $\mu_{a,e}$, we consider a smaller value, i.e., $\mu_{a,e} = 1$, and a larger value, i.e., $\mu_{a,e} = 3$, in this sensitivity analysis.

The resulting Pareto fronts are shown in Figure 7b, c, while the corresponding performance indicators are summarized in Table 14. The objective minima for each variant are shown in Table A3 in Appendix C. For both values of $\mu_{a,e}$, the same qualitative conclusions hold. Introducing frequency optimization increases both the quality and diversity of the obtained Pareto fronts, while the inclusion of parking-fee optimization yields a substantially larger improvement. This is confirmed quantitatively by the HV and IGD values. For the lower value $\mu_{a,e} = 1$, public transport is perceived as relatively more attractive. In this case, the TNDFSP and TNDFP achieve 37.6% and 37.2% of the TNDFPSP HV, indicating that parking-fee optimization remains the main source of improvement. Increasing the mode-specific constant to $\mu_{a,e} = 3$ makes transit less attractive and slightly increases the value of frequency optimization relative to the TNDFP. However, the improvement remains modest compared to the gains achieved by also optimizing parking fees.

6.4.3 | VoT

We continue with a sensitivity analysis on the VoT parameter values as given in Table 3 in Section 5.2. Based on the literature, a value of 10 is a reasonable approximation for all VoT values. However, Wardman [90, 103] suggests that VoT values for waiting for transit, walking, and searching for parking might be larger (roughly a factor of 1.5) than VoT values for in-vehicle travel time. Cycling was not included in these studies, but it is reasonable to assume that cycling is valued similarly to walking. We therefore look at the case where $\text{VoT}_{\text{search}}$, VoT_{walk} , $\text{VoT}_{\text{cycle}}$ and VoT_{wait} are equal to fifteen.

The resulting Pareto front is shown in Figure 7d, while the HV and IGD values are reported in Table 14. The objective minima are displayed in Table A4 in Appendix C. The same qualitative conclusions remain valid. A higher valuation of searching, walking, cycling, and waiting times reduces the sensitivity of generalized travel costs to the considered decision variables, resulting in somewhat less diversity in terms of car-user outcomes. Nevertheless, the TNDFPSP continues to produce substantially better Pareto fronts than both alternative variants. The TNDFSP and TNDFP attain only 36.5% and 33.8% of the TNDFPSP HV, respectively, while also exhibiting considerably larger IGD values. Hence, the added value of jointly optimizing parking fees and transit supply remains robust with respect to plausible variations in VoT assumptions.

6.4.4 | Parking Duration

The parking duration directly scales the contribution of parking fees to the generalized travel cost and is therefore expected to be particularly influential. To investigate this effect, we consider both a shorter parking duration ($T_{\text{park}} = 1$) and a longer parking duration ($T_{\text{park}} = 3$).

The resulting Pareto fronts are shown in Figure 7e,f, while Table 14 summarizes the corresponding HV and IGD values. The objective minima for the three variants are provided in Table A5 in Appendix C. Among all considered sensitivity parameters,

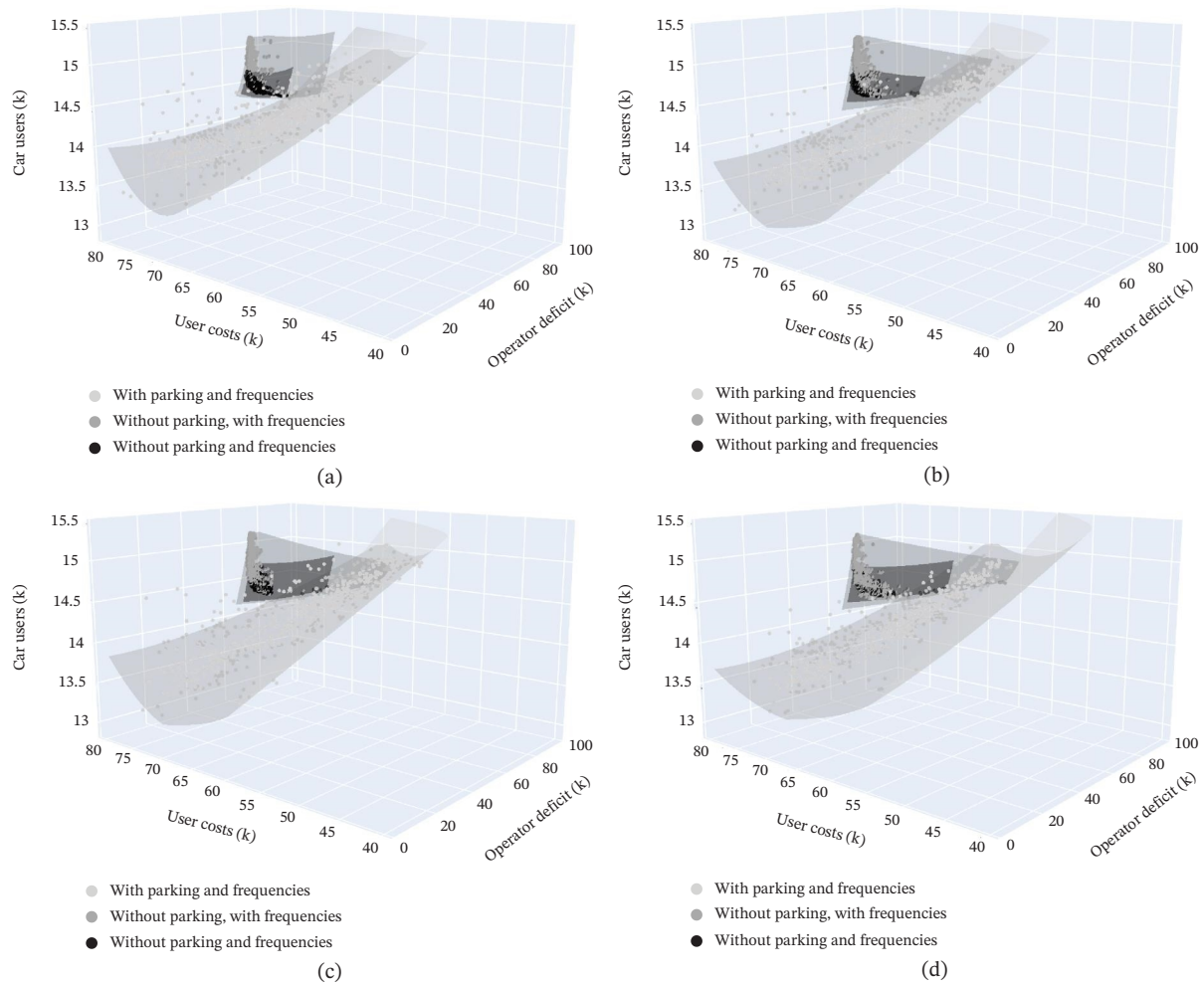


FIGURE 6 | Pareto fronts for the three problem variants: with and without frequency and parking fee optimization. (a) Four lines. (b) Six lines. (c) Seven lines. (d) Eight lines.

parking duration has the largest impact on the obtained Pareto fronts. This is not surprising, as the parking fee itself is a decision variable and parking duration directly scales its effect on the generalized travel cost of car users. Even for shorter parking durations, where parking costs play a smaller role, the TNDFPSP continues to substantially outperform the alternative variant. The TNDFPSP and TNDFP attain only 50.5% and 47.6% of the TNDFPSP HV, respectively, while the latter also achieves the smallest IGD value. Hence, meaningful gains from parking-fee optimization remain possible even when parking durations are relatively short. When the parking duration is increased to $T_{\text{park}} = 3$, the effect of parking pricing becomes considerably stronger. As a result, the differences between the TNDFPSP and the two alternative variants become more pronounced. Nevertheless, the qualitative conclusions remain unchanged across both parking-duration scenarios: jointly optimizing transit routes, frequencies, and parking fees consistently yields the best-performing Pareto front.

6.4.5 | Operator Costs per Kilometer

We proceed with a sensitivity analysis for the value of the operator costs per kilometer, O , which we chose to be twenty in Section 5.2. Again, as the operator deficit as given in Equation

(13) is linear in O , we consider a smaller value, i.e., $O = 10$, and a larger value, i.e., $O = 30$, in this sensitivity analysis.

The resulting Pareto fronts are shown in Figure 7g,h, while the corresponding HV and IGD values are provided in Table 14. The objective minima for each variant are reported in Table A6 in Appendix C. As expected, varying O primarily affects the operator-deficit objective, while the overall structure of the obtained Pareto fronts remains largely unchanged. For both values considered, the qualitative conclusions remain identical: frequency optimization provides a modest improvement over the TNDFP, while the inclusion of parking-fee optimization yields substantially larger gains. This is reflected in the HV percentages, where the TNDFPSP attains 35.8% and 38.0% of the TNDFPSP HV for $O = 10$ and $O = 30$, respectively, whereas the corresponding values for the TNDFP are 33.4% and 34.9%. However, these differences remain relatively small compared to the gap between the TNDFPSP and the two alternative variants. The TNDFPSP consistently achieves the highest HV and lowest IGD values, indicating that the main conclusions are robust with respect to uncertainty in operating-cost estimates.

6.4.6 | Summary

Overall, the sensitivity analyses demonstrate that the main conclusions of this study are robust with respect to the considered

TABLE 9 | Hypervolume (HV) and inverted generational distance (IGD) for the three problem variants.

#lines	Variant	HV	HV (%)	IGD
4	With parking and frequencies	0.754	100.0	0.010
	Without parking, with frequencies	0.283	37.5	0.255
	Without parking and frequencies	0.270	35.8	0.258
6	With parking and frequencies	0.760	100.0	0.022
	Without parking, with frequencies	0.279	36.7	0.244
	Without parking and frequencies	0.274	36.0	0.258
7	With parking and frequencies	0.757	100.0	0.023
	Without parking, with frequencies	0.281	37.1	0.243
	Without parking and frequencies	0.270	35.6	0.265
8	With parking and frequencies	0.765	100.0	0.024
	Without parking, with frequencies	0.299	39.1	0.233
	Without parking and frequencies	0.282	36.9	0.252

Note: Bold values indicate the best result for each line-setting variant (highest HV and lowest IGD).

TABLE 10 | Illustrative total CO₂ emissions (kilograms) for CU-MIN-(FP) solutions with four lines.

Solution	CU-MIN	CU-MIN-F	CU-MIN-FP
Total CO ₂ (kg)	18,427	18,401	17,807

parameter variations. While the magnitude of the improvements varies across scenarios, particularly for parking duration, the qualitative ranking of the three problem variants remains unchanged. The results consistently show that the gains obtained by adding parking-fee optimization on top of frequency optimization are substantially larger than those obtained by introducing frequency optimization alone. The objective minima reported in Appendix C provide additional support for these findings.

6.5 | Validation and Robustness Analyses

The previous subsection demonstrated that the main conclusions are robust with respect to variations in key model parameters. We now consider two additional aspects related to the validity of the proposed framework. First, we assess whether neglecting transit crowding effects is reasonable for the considered scenarios by examining transit loads across the obtained Pareto-optimal solutions. Second, we investigate the convergence of the multimodal assignment procedure and confirm that the adopted number of iterations is sufficient under both normal and highly congested conditions.

6.5.1 | Transit Crowding Assessment

The transit assignment model employed in this study does not explicitly account for in-vehicle crowding effects. To assess whether this assumption is reasonable for the considered experiments, we perform a post hoc crowding assessment of the obtained TNDFPSP solutions. For each Pareto-optimal solution, passenger loads are calculated for all transit links. Because frequencies are decision variables, crowding levels depend not only on passenger demand but also on the deployed service frequencies. We therefore compute the passenger load per vehicle by dividing the passenger flow on a transit link by the corresponding service frequency.

Table 15 summarizes the resulting crowding statistics. For each Pareto-optimal solution, we determine the most heavily loaded transit link and subsequently report aggregate statistics across all solutions.

For the base scenario with four lines, crowding is limited. The average maximum vehicle load equals approximately 12 passengers, while even the most heavily loaded observed vehicle carries only 26 passengers. These values are well below typical bus capacities, suggesting that omitting crowding effects is unlikely to substantially affect the obtained Pareto fronts.

To further evaluate the robustness of this conclusion, we also consider the sensitivity scenario with $T_{\text{park}} = 3$, which generates the largest modal shift toward public transport. As expected, vehicle loads increase substantially. Nevertheless, average maximum loads remain moderate, and even the largest observed load remains below typical vehicle capacities. The 95th percentile of the maximum vehicle loads equals 51.63 passengers, while the most heavily loaded service carries 71.69 passengers per vehicle. Although these values do not directly imply capacity violations, they suggest that crowding may become noticeable on a small number of heavily used vehicles. Consequently, crowding effects are unlikely to affect the conclusions of the present study substantially but may become increasingly relevant in scenarios that generate even larger shifts toward public transport.

6.5.2 | Multimodal Assignment Convergence

As discussed in Section 3.3, the multimodal assignment procedure alternates between mode-choice and traffic-assignment calculations. In the computational experiments, two iterations of this procedure were selected, as preliminary analyses suggested that additional iterations have little influence on the resulting demand patterns.

To verify this assumption, we first consider the base scenario with four transit lines. Specifically, we compare the modal demands obtained after the second and third multimodal assignment iterations. The relative demand change is computed as the difference between the demands after the third and second iterations, divided by the total demand, and multiplied by 100 to convert to percentages. The results are summarized in Table 16.

For the base scenario, the observed changes are extremely small relative to the total demand, indicating that the multimodal assignment is already close to convergence after two iterations. Consequently, additional iterations are unlikely to substantially affect the resulting Pareto fronts or objective values.

To investigate whether convergence becomes more challenging under stronger congestion effects, we repeat the same analysis for the congestion scenario introduced in Section 6.4 ($c_a = 1500$). Since congestion increases the interaction between mode choice

TABLE 11 | Comparison of CU-MIN and CU-MIN-F with solutions for four lines with a similar number of car users obtained by solving the full TNDFPSP.

Solution	User costs	Car users	Operator deficit	Gain UC (%)	Gain OD (%)
CU-MIN	57,982	14,795	21,597	—	—
TNDFPSP (low UC)	52,215	14,794	2666	9.96	87.66
TNDFPSP (low OD)	57,318	14,793	1483	1.15	93.13
CU-MIN-F	58,126	14,712	38,051	—	—
TNDFPSP (low UC)	51,299	14,711	6984	11.75	81.64
TNDFPSP (low OD)	52,311	14,712	3980	9.99	89.54

Note: Percentage changes are relative to CU-MIN or CU-MIN-F.

TABLE 12 | Impact of the expansion operator on minimal user costs (UC-MIN-F, UC-MIN-FP) and minimal car users (CU-MIN, CU-MIN-F, CU-MIN-FP).

#lines	Solution	Baseline	With operator	Improvement (%)
4	UC-MIN-F	56,100	56,100	0.00
	UC-MIN-FP	41,265	40,473	1.91
	CU-MIN	14,800	14,795	0.03
	CU-MIN-F	14,791	14,712	0.53
	CU-MIN-FP	13,478	13,026	3.35
6	UC-MIN-F	56,109	56,109	0.00
	UC-MIN-FP	41,314	40,546	1.86
	CU-MIN	14,802	14,739	0.43
	CU-MIN-F	14,708	14,658	0.34
	CU-MIN-FP	13,269	12,892	2.84
7	UC-MIN-F	56,101	56,101	0.00
	UC-MIN-FP	41,093	40,564	1.29
	CU-MIN	14,804	14,724	0.54
	CU-MIN-F	14,694	14,644	0.34
	CU-MIN-FP	13,226	12,824	3.03
8	UC-MIN-F	56,275	56,275	0.00
	UC-MIN-FP	41,541	40,550	2.38
	CU-MIN	14,726	14,692	0.23
	CU-MIN-F	14,653	14,624	0.20
	CU-MIN-FP	13,180	12,795	2.93

TABLE 13 | Parameters included in the sensitivity analysis, their role in the model, and rationale for inclusion.

Parameter	What it affects	Rationale for inclusion
Road capacity (c_a)	Determines the level of congestion experienced by car users and therefore affects car travel times, modal split, and parking-fee effectiveness.	The base network exhibits moderate congestion. Assessing lower capacities allows us to evaluate whether the conclusions remain valid under more congested traffic conditions.
Mode-specific constant for transit ($\mu_{a,e}$)	Baseline discomfort of transit relative to car; directly shifts the attractiveness of public transport in the mode-choice model.	Literature values vary and empirical evidence is limited; the parameter strongly influences modal split.
Value of Time (VoT) parameters	Weighting of time components (searching, walking, cycling, waiting) in generalized cost functions for both car and transit.	VoT estimates differ by context and population; they affect the relative importance of travel time and monetary costs in traveller decisions.
Parking duration (T_{park})	Determines the total parking cost incurred by car users through the parking-fee component of the generalized cost.	Parking duration directly scales the impact of parking fees and therefore influences the effectiveness of parking-pricing policies.

(Continues)

TABLE 13 | (Continued)

Parameter	What it affects	Rationale for inclusion
Operator cost per km (O)	Determines operator deficit; impacts the trade-off between service provision and financial sustainability.	Real-world operating costs are uncertain and context-dependent; the parameter influences recommendations regarding transit investment and operation.

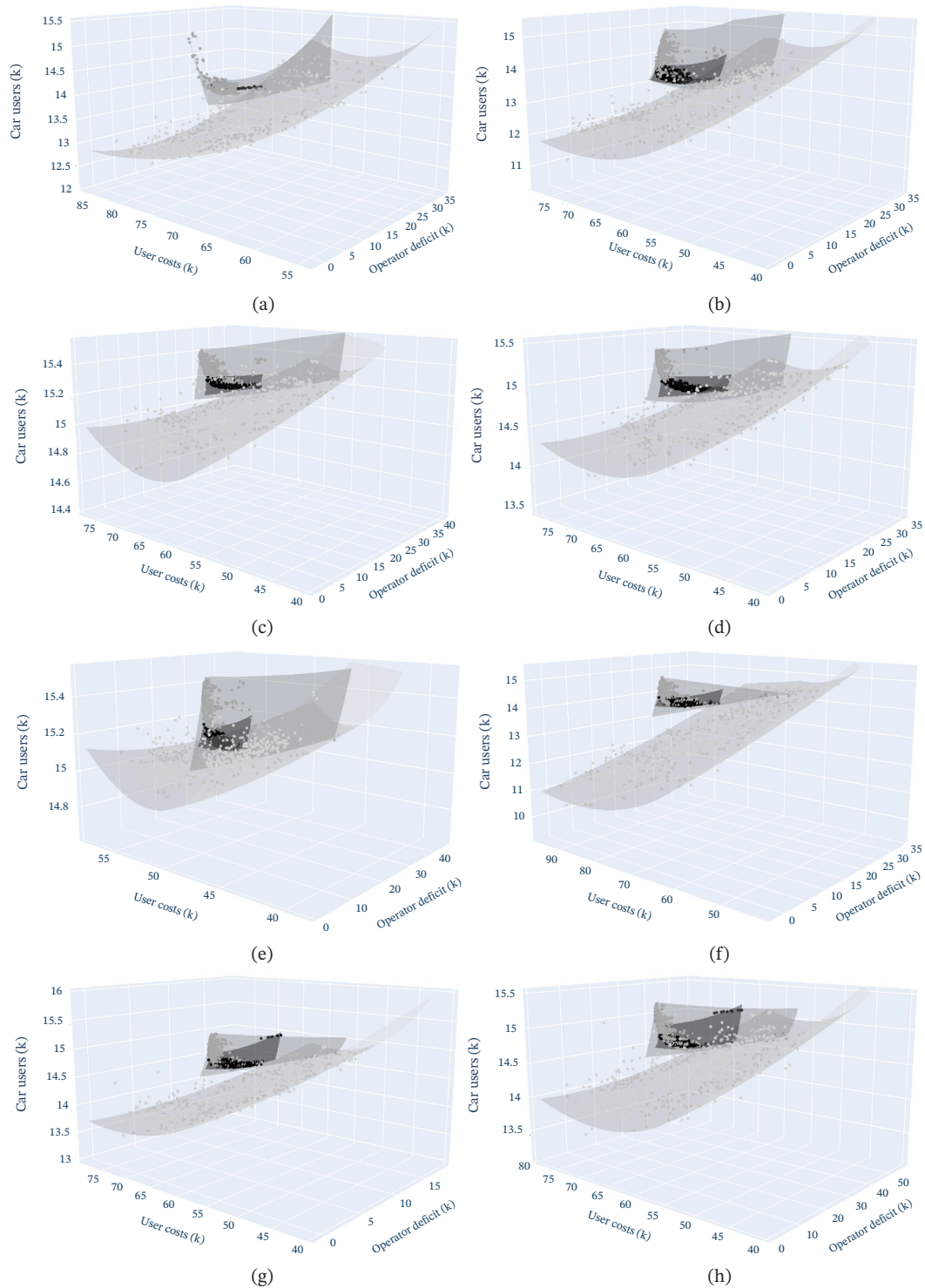


FIGURE 7 | Pareto fronts for the sensitivity analysis. The meaning of each color is identical to that of Figure 6: black represents the TNDP, dark gray the TNDFSP, and light gray the TNDFPSP. (a) Road capacity $c_a = 1500$. (b) $\mu_{a,e} = 1$. (c) $\mu_{a,e} = 3$. (d) $VoT_{search} = VoT_{walk} = VoT_{cycle} = VoT_{wait} = 15$. (e) $T_{park} = 1$. (f) $T_{park} = 3$. (g) $O = 10$. (h) $O = 30$.

TABLE 14 | Hypervolume (HV) and inverted generational distance (IGD) for the sensitivity analyses.

Scenario	Variant	HV	HV (%)	IGD
$c_a = 1500$	With parking and frequencies	0.833	100.0	0.000
	Without parking, with frequencies	0.310	37.2	0.291
	Without parking and frequencies	0.277	33.3	0.314
$\mu_{a,e} = 1$	With parking and frequencies	0.758	100.0	0.000
	Without parking, with frequencies	0.285	37.6	0.278
	Without parking and frequencies	0.282	37.2	0.277
$\mu_{a,e} = 3$	With parking and frequencies	0.746	100.0	0.001
	Without parking, with frequencies	0.262	35.2	0.265
	Without parking and frequencies	0.246	33.0	0.273
VoT	With parking and frequencies	0.745	100.0	0.026
	Without parking, with frequencies	0.272	36.5	0.235
	Without parking and frequencies	0.252	33.8	0.259
$T_{\text{park}} = 1$	With parking and frequencies	0.797	100.0	0.0175
	Without parking, with frequencies	0.402	50.5	0.185
	Without parking and frequencies	0.380	47.6	0.206
$T_{\text{park}} = 3$	With parking and frequencies	0.733	100.0	0.000
	Without parking, with frequencies	0.202	27.6	0.340
	Without parking and frequencies	0.199	27.1	0.340
$O = 10$	With parking and frequencies	0.772	100.0	0.001
	Without parking, with frequencies	0.277	35.8	0.301
	Without parking and frequencies	0.258	33.4	0.309
$O = 30$	With parking and frequencies	0.748	100.0	0.004
	Without parking, with frequencies	0.284	38.0	0.246
	Without parking and frequencies	0.261	34.9	0.261

Note: Bold values indicate the best result for each line-setting variant (highest HV and lowest IGD).

TABLE 15 | Transit crowding statistics for all TNDFPSP solutions.

Scenario	Mean maximum load	Median maximum load	95th percentile	Maximum load
Base case (4 lines)	11.72	11.16	21.46	26.24
$T_{\text{park}} = 3$	26.66	24.72	51.63	71.69

Note: Vehicle load is defined as passenger flow divided by service frequency.

TABLE 16 | Relative changes in modal demand between the second and third multimodal assignment iterations.

Scenario	Mean (%)	Median (%)	95th percentile (%)	Maximum (%)
Base case (4 lines)	0.0011	0.0011	0.0013	0.0032
Congested ($c_a = 1500$)	0.200	0.256	0.294	0.407

and traffic conditions, larger changes between successive iterations are expected. As shown in Table 16, the relative demand changes are indeed larger than in the base case. Nevertheless, even in this congested scenario, the average difference remains approximately 0.2%, while the maximum observed difference is approximately 0.4%. These values suggest that the adopted number of multimodal assignment iterations is sufficient for the scenarios considered in this study.

Overall, the results provide additional confidence in the computational procedure. However, for scenarios with substantially higher congestion levels, stronger parking constraints, or larger

modal shifts, increasing the number of multimodal assignment iterations may be desirable to ensure a fully converged modal split.

6.6 | Computational Analysis and Scalability

All experiments were conducted on the Snellius supercomputer, utilizing the “fat nodes” partition, which offers large memory capacity but relatively modest per-core performance [104]. Up to sixteen independent algorithm runs could be executed in parallel on a single fat node, each using 1/8 of the node’s resources. The system allowed for roughly five such jobs to be submitted simultaneously, subject to queue and budget constraints. This

parallelization strategy was chosen to maximize throughput within the available computational budget and infrastructure limitations.

A typical experiment, consisting of 100 iterations with a population size of 500, required approximately 1.5 days of wall-clock time. Although the fat nodes are not optimized for single-thread performance, their large memory makes them well-suited for running many experiments concurrently. The total runtime on a relatively small benchmark network may appear long, but this is primarily due to the chosen computational strategy and infrastructure. In this study, the available resources were used to maximize the number of independent algorithm runs executed in parallel, rather than parallelizing components within a single run.

It should be noted that the car assignment step constitutes the majority of the computational effort. In particular, the Frank–Wolfe algorithm used to solve the road assignment problem is executed repeatedly for each OD-pair during the optimization process and therefore largely determines the overall runtime. Since the assignment calculations for different OD-pairs can be performed independently, significant speedups can be achieved through parallelization of this component.

Consequently, substantial reductions in computation time can be achieved through within-run parallelization, in contrast to the present study, where parallelization was applied primarily across independent algorithm runs. The reported run-times should therefore not be interpreted as an inherent limitation of the proposed TNDFPSP framework but rather as a consequence of the chosen experimental setup and available computational resources. For larger and more realistic urban networks, efficient parallel implementations of the traffic assignment procedure are expected to provide significant speedups and represent a promising direction for improving scalability. Nevertheless, implementing parallel traffic-assignment procedures for large-scale applications is beyond the scope of this study. The focus here is on establishing the value of the proposed TNDFPSP framework; improving computational scalability through parallelization is a natural next step that could substantially improve scalability and facilitate future applications to larger, real-world networks.

7 | Conclusion and Discussion

In this paper, we introduced and studied the TNDFPSP, a novel multiobjective formulation that jointly optimizes transit routes, service frequencies, and parking fees. This formulation explicitly incorporates modal shift as an optimization objective alongside minimizing generalized user costs and operator deficit. By integrating carrot-and-stick measures within a unified framework, the TNDFPSP enables systematic analysis of policy trade-offs between encouraging public transport use and discouraging car use to achieve a more sustainable mobility system.

To solve this problem, we proposed an NSGA-II-based algorithm tailored to the TNDFPSP. We extended existing crossover and mutation operators to handle parking fees as decision variables, applied a repair strategy to ensure network connectivity, and introduced an expansion operator that exploits relationships between decision variables and objectives to improve the Pareto front. For solution evaluation, our multimodal traffic assignment model

further advances existing methods by incorporating parking fees as decision variables and park search time, as well as by introducing generalized cost functions for consistent evaluation across modes.

Empirical results on Mandl's network demonstrate that the TNDFPSP framework enables substantial improvements across all objectives. Notably, our results show that jointly optimizing transit routes, frequencies, and parking fees can yield a significant reduction in CO₂ emissions (3.4%) compared to considering only transit routes and frequencies (0.14%), while also providing policymakers with better trade-off solutions. Finally, the proposed expansion operator proved effective in extending the Pareto front, particularly for the full TNDFPSP, offering additional solutions beyond those generated by standard evolutionary steps.

A key next step is the application of the TNDFPSP framework to larger, real-world networks. Such applications would require addressing computational challenges, particularly through parallelization of the traffic-assignment procedure and the use of high-performance computing resources. While these computational improvements are beyond the scope of the present study, they offer a promising route toward improving scalability without altering the underlying methodology.

The modeling assumptions adopted throughout this study follow common practice in strategic transit network design. Some assumptions and parameter choices, such as symmetric OD demands, were selected specifically for Mandl's benchmark network but do not restrict the general applicability of the framework. Other assumptions simplify aspects of real-world complexity, including uniform fare structures, the absence of transit crowding effects, deterministic travel behavior, and the assumption that transit services operate independently of road congestion. Similarly, operating economics are represented at an aggregate level through uniform line-cost assumptions and a simplified fleet-cost formulation, which may not capture all cost differences observed in practice. Nevertheless, the sensitivity, crowding, and convergence analyses suggest that the main qualitative conclusions remain unchanged: jointly optimizing carrot-and-stick measures is consistently more effective than addressing either in isolation.

At the same time, these simplifying assumptions indicate several opportunities for further extending the framework. These include incorporating heterogeneous user groups and fare structures, in-vehicle crowding, stochastic mode and route choice behavior, and transit services operating in mixed traffic. Such additions could also improve the representation of traveler heterogeneity and support the analysis of equity and distributional impacts across different user groups. The current framework assumes fixed OD demand and therefore captures the impact of parking fees primarily through mode choice. Relaxing this assumption through elastic travel demand would allow parking policies to influence not only modal split but also trip generation, destination choice, trip chaining, and shared-mobility or carpooling decisions. Another extension would be to explicitly account for parking-fee revenues. While parking fees are treated in this study primarily as a policy instrument to influence travel behavior, they could also be incorporated into integrated funding mechanisms in which parking revenues partially compensate public transport operating costs or contribute to broader system-level welfare objectives. Finally, alternative objective formulations may be considered, such as minimizing vehicle-kilometers traveled or emissions instead of the number of car users.

Acknowledgments

Some experiments in this paper were conducted on the Dutch national e-infrastructure with the support of the SURF Cooperative with grant number EINF-9258.

Funding

The research was supported by the Dutch Research Council (NWO) through the Gravitation project NETWORKS under grant number 024.002.003 and the Perspectief Project XCARCITY with file number P21-08.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

References

1. CBS, "Hoeveel Wordt Er Met Het Openbaar Vervoer Gereisd?" (2022), <https://www.cbs.nl/nl-nl/visualisaties/verkeer-en-vervoer/personen/openbaar-vervoer>.
2. A. Ceder and N. H. M. Wilson, "Bus Network Design," *Transportation Research Part B: Methodological* 20, no. 4 (1986): 331–344, [https://doi.org/10.1016/0191-2615\(86\)90047-0](https://doi.org/10.1016/0191-2615(86)90047-0).
3. T. L. Magnanti and R. T. Wong, "Network Design and Transportation Planning: Models and Algorithms," *Transportation Science* 18, no. 1 (1984): 1–55, <https://doi.org/10.1287/trsc.18.1.1>.
4. F. Lopez, "Integrating Network Design and Frequency Setting in Public Transportation Networks: A Survey," *Statistics and Operations Research Transactions* 38 (2014): 181–214, <https://www.raco.cat/index.php/SORT/article/view/284041>.
5. R. Balcombe, R. Mackett, N. Paulley, et al., *The Demand for Public Transport: A Practical Guide* (Transport Research Laboratory, 2004).
6. B. De Borger and S. Proost, "A Political Economy Model of Road Pricing," *Journal of Urban Economics* 71, no. 1 (2012): 79–92, <https://doi.org/10.1016/j.jue.2011.08.002>.
7. D. Shoup, *High Cost of Free Parking* (Routledge, 2018).
8. B. Montella, M. Gallo, and L. D'Acerno, "Multimodal Network Design Problems," *Advances in Transport* 5 (2000): 405–414, <https://doi.org/10.2495/UT990381>.
9. D. P. Piatkowski, W. E. Marshall, and K. J. Krizek, "Carrots Versus Sticks: Assessing Intervention Effectiveness and Implementation Challenges for Active Transport," *Journal of Planning Education and Research* 39, no. 1 (2019): 50–64, <https://doi.org/10.1177/0739456X17715306>.
10. E. Inci, "A Review of the Economics of Parking," *Economics of Transportation* 4, no. 1 (2015): 50–63, <https://doi.org/10.1016/j.ecotra.2014.11.001>.
11. J. Durán-Micco and P. Vansteenwegen, "A Survey on the Transit Network Design and Frequency Setting Problem," *Public Transport* 14, no. 1 (2022): 155–190, <https://doi.org/10.1007/s12469-021-00284-y>.
12. Y. Liao, J. Gil, R. H. M. Pereira, S. Yeh, and V. Verendel, "Disparities in Travel Times Between Car and Transit: Spatiotemporal Patterns in Cities," *Scientific Reports* 10, no. 1 (2020): 4056, <https://doi.org/10.1038/s41598-020-61077-0>.
13. B. Possel, L. J. J. Wismans, E. C. Van Berkum, and M. C. J. Bliemer, "The multi-objective Network Design Problem Using Minimizing Externalities as Objectives: Comparison of a Genetic Algorithm and Simulated Annealing Framework," *Transportation* 45, no. 2 (2018): 545–572, <https://doi.org/10.1007/s11116-016-9738-y>.
14. M. T. M. Emmerich and A.H. Deutz, "A Tutorial on Multiobjective Optimization: Fundamentals and Evolutionary Methods," *Natural Computing* 17, no. 3 (2018): 585–609, <https://doi.org/10.1007/s11047-018-9685-y>.
15. Y. Zhou, C. Cao, and Z. Feng, "Optimization of Multimodal Discrete Network Design Problems Based on Super Networks," *Applied Sciences* 11, no. 21 (2021): 10143, <https://doi.org/10.3390/app11210143>.
16. H. Wang, W. H. K. Lam, X. Zhang, and H. Shao, "Sustainable Transportation Network Design With Stochastic Demands and Chance Constraints," *International Journal of Sustainable Transportation* 9, no. 2 (2015): 126–144, <https://doi.org/10.1080/15568318.2012.740147>.
17. L. Zhang, H. Yang, D. Wu, and D. Wang, "Solving a Discrete Multimodal Transportation Network Design Problem," *Transportation Research Part C: Emerging Technologies* 49 (2014): 73–86, <https://doi.org/10.1016/j.trc.2014.10.008>.
18. M. Metin Mutlu, I. Cihan Aksoy, and Y. Alver, "Transit Frequency Optimization in Bi-Modal Networks Using Differential Evolution Algorithm," *Teknik Dergi* 33, no. 5 (2022): 12601–12616, <https://doi.org/10.18400/tekderg.935950>.
19. L. Qi, S. Samaranayake, and S. Banerjee, "Multimodal Mobility Systems: Joint Optimization of Transit Network Design and Pricing," in *Proceedings of the ACM/IEEE 12th International Conference on Cyber-Physical Systems, ICCPS '21* (ACM, 2021), 121–131, <https://doi.org/10.1145/3450267.3450540>.
20. B. Basciftci and P. Van Hentenryck, "Capturing Travel Mode Adoption in Designing On-Demand Multimodal Transit Systems," *Transportation Science* 57, no. 2 (2023): 351–375, <https://doi.org/10.1287/trsc.2022.1184>.
21. J. Zhao, S. Sun, and O. Cats, "Joint Optimisation of Regular and demand-responsive Transit Services," *Transportmetrica A: Transportation Science* 19, no. 2 (2023): <https://doi.org/10.1080/23249935.2021.1987580>.
22. H. K. R. F. Pinto, M. F. Hyland, H. S. Mahmassani, and I. Ömer Verbas, "Joint Design of Multimodal Transit Networks and Shared Autonomous Mobility Fleets," *Transportation Research Part C: Emerging Technologies* 113 (2020): 2–20, <https://doi.org/10.1016/j.trc.2019.06.010>.
23. L. Caggiani, A. Colovic, and M. Ottomanelli, "An Equality-Based Model for Bike-Sharing Stations Location in Bicycle-Public Transport Multimodal Mobility," *Transportation Research Part A: Policy and Practice* 140 (2020): 251–265, <https://doi.org/10.1016/j.tra.2020.08.015>.
24. S. Xanthopoulos, M. van der Tuin, S. Sharif Azadeh, G. H. de A. Correia, N. van Oort, and M. Snelder, "Optimization of the Location and Capacity of Shared Multimodal Mobility Hubs to Maximize Travel Utility in Urban Areas," *Transportation Research Part A: Policy and Practice* 179 (2024): <https://doi.org/10.1016/j.tra.2023.103934>.
25. T. Brands and E. C. van Berkum, "Performance of a Genetic Algorithm for Solving the Multi-Objective, Multimodal Transportation Network Design Problem," *International Journal of Transportation* 2, no. 1 (2014): 1–20, <https://doi.org/10.14257/ijt.2014.2.1.01>.
26. A. Mohsena Haque, C. Brakewood, S. Rezaei, and A. Khojandi, "A Literature Review on Park-and-Rides," *Journal of Transport and Land Use* 14, no. 1 (2021): 1039–1060, <https://doi.org/10.5198/jtlu.2021.1923>.
27. J. Ortega, J. Tóth, and T. Péter, "Planning a Park and Ride System: A Literature Review," *Future Transportation* 1, no. 1 (2021): 82–98, <https://doi.org/10.3390/futuretransp1010006>.
28. J. A. Mesa, F. A. Ortega, M. A. Pozo, and R. Piedra de-la Cuadra, "Assessing the Effectiveness of Park-and-Ride Facilities on Multimodal Networks in Smart Cities," *Journal of the Operational Research Society* 73, no. 3 (2022): 576–586, <https://doi.org/10.1080/01605682.2020.1854628>.
29. S. Rezaei, A. Khojandi, A. M. Haque, C. Brakewood, M. Jin, and C. R. Cherry, "Park-And-Ride Facility Location Optimization: A Case Study for Nashville, Tennessee," *Transportation Research Interdisciplinary Perspectives* 13 (2022): <https://doi.org/10.1016/j.trip.2022.100578>.

30. L. D'Acierno, M. Gallo, and B. Montella, "Optimisation Models for the Urban Parking Pricing Problem," *Transport Policy* 13, no. 1 (2006): 34–48, <https://doi.org/10.1016/j.tranpol.2005.08.001>.
31. J. Cavadas and A. P. Antunes, "An Optimization Model for Integrated Transit-Parking Policy Planning," *Transportation* 46, no. 5 (2019): 1867–1891, <https://doi.org/10.1007/s11116-018-9905-4>.
32. A. Loder, M. C. J. Bliemer, and K. W. Axhausen, "Optimal Pricing and Investment in a Multi-Modal City—Introducing a Macroscopic Network Design Problem Based on the MFD," *Transportation Research Part A: Policy and Practice* 156 (2022): 113–132, <https://doi.org/10.1016/j.tra.2021.11.026>.
33. M. Kim, S.-Y. Kho, and D.-K. Kim, "A Transit Route Network Design Problem Considering Equity," *Sustainability* 11, no. 13 (2019): 2019, <https://doi.org/10.3390/su11133527>.
34. S. J. Park, S. Kang, Y.-J. Byon, and S.-Y. Kho, "Multiobjective Approach to the Transit Network Design Problem With Variable Demand Considering Transit Equity," *Journal of Advanced Transportation* 2022 (2022): 2022–17, <https://doi.org/10.1155/2022/5887985>.
35. J. Durán, L. Pradenas, and V. Parada, "Transit Network Design With Pollution Minimization," *Public Transport* 11, no. 1 (2019): 189–210, <https://doi.org/10.1007/s12469-019-00200-5>.
36. J. Durán-Micco, E. Vermeir, and P. Vansteenwegen, "Considering Emissions in the Transit Network Design and Frequency Setting Problem With a Heterogeneous Fleet," *European Journal of Operational Research* 282, no. 2 (2020): 580–592, <https://doi.org/10.1016/j.ejor.2019.09.050>.
37. I. A. M. Sadeghi, M. Ghaznavi, and M. Ghaznavi, "A Cuckoo Search Based Approach to Design Sustainable Transit Network," *Transportation Letters* 13, no. 9 (2021): 635–648, <https://doi.org/10.1080/19427867.2020.1750767>.
38. A. De-Los-Santos, D. Canca, and E. Barrena, "Mathematical Formulations for the Bimodal Bus-Pedestrian Social Welfare Network Design Problem," *Transportation Research Part B: Methodological* 145 (2021): 302–323, <https://doi.org/10.1016/j.trb.2021.01.010>.
39. P. Heyken Soares, "Zone-Based Public Transport Route Optimisation in an Urban Network," *Public Transport* 13, no. 1 (2021): 197–231, <https://doi.org/10.1007/s12469-020-00242-0>.
40. D. Canca, A. De-Los-Santos, G. Laporte, and J. A. Mesa, "An Adaptive Neighborhood Search Metaheuristic for the Integrated Railway Rapid Transit Network Design and Line Planning Problem," *Computers & Operations Research* 78 (2017): 1–14, <https://doi.org/10.1016/j.cor.2016.08.008>.
41. M. H. Almasi, Y. Oh, S. Ali, Y.-J. Byon, and S. Kang, "Urban Transit Network Optimization Under Variable Demand With Single and Multi-Objective Approaches Using Metaheuristics: The Case of Daejeon, Korea," *International Journal of Sustainable Transportation* 15, no. 5 (2021): 386–406, <https://doi.org/10.1080/15568318.2020.1821414>.
42. P. W. Bonsall and I. A. Palmer, "Modelling Drivers' Car Parking Behaviour Using Data From a Travel Choice Simulator," *Transportation Research Part C: Emerging Technologies* 12, no. 5 (2004): 321–347, <https://doi.org/10.1016/j.trc.2004.07.013>.
43. J. G. Wardrop, "Some Theoretical Aspects of Road Traffic Research," *Proceedings of the Institution of Civil Engineers* 1, no. 3 (1952): 325–362, <https://doi.org/10.1680/jipeds.1952.11259>.
44. M. Frank and P. Wolfe, "An Algorithm for Quadratic Programming," *Naval Research Logistics Quarterly* 3, no. 1-2 (1956): 95–110, <https://doi.org/10.1002/nav.3800030109>.
45. H. Zhao, W. A. Xu, and R. Jiang, "The Memetic Algorithm for the Optimization of Urban Transit Network," *Expert Systems With Applications* 42, no. 7 (2015): 3760–3773, <https://doi.org/10.1016/j.eswa.2014.11.056>.
46. H. Spiess and M. Florian, "Optimal Strategies: A New Assignment Model for Transit Networks," *Transportation Research Part B: Methodological* 23, no. 2 (1989): 83–102, [https://doi.org/10.1016/0191-2615\(89\)90034-9](https://doi.org/10.1016/0191-2615(89)90034-9).
47. M. Hadi Baaj and H. S. Mahmassani, "TRUST: A LISP Program for the Analysis of Transit Route Configurations," *Transportation Research Record* 1283 (1990): 125–135, <http://onlinepubs.trb.org/Onlinepubs/trr/1990/1283/1283-013.pdf>.
48. M. Cepeda, R. Cominetti, and M. Florian, "A Frequency-Based Assignment Model for Congested Transit Networks With Strict Capacity Constraints: Characterization and Computation of Equilibria," *Transportation Research Part B: Methodological* 40, no. 6 (2006): 437–459, <https://doi.org/10.1016/j.trb.2005.05.006>.
49. Y. Hamdouch and S. Lawphongpanich, "Schedule-Based Transit Assignment Model With Travel Strategies and Capacity Constraints," *Transportation Research Part B: Methodological* 42, no. 7 (2008): 663–684, <https://doi.org/10.1016/j.trb.2007.11.005>.
50. M. Owais and A. S. Ahmed, "Frequency Based Transit Assignment Models: Graph Formulation Study," *IEEE Access* 10 (2022): 62991–63003, <https://doi.org/10.1109/ACCESS.2022.3182046>.
51. M. Balmer, M. Rieser, K. Meister, D. Charypar, N. Lefebvre, and K. Nagel, "MATSim-T: Architecture and Simulation Times," in *Multi-Agent Systems for Traffic and Transportation Engineering* (IGI Global Scientific Publishing, 2009), 57–78.
52. M. Wahba and A. Shalaby, "Learning-Based Framework for Transit Assignment Modeling Under Information Provision," *Transportation* 41, no. 2 (2014): 397–417, <https://doi.org/10.1007/s11116-013-9510-5>.
53. O. Cats and J. West, "Learning and Adaptation in Dynamic Transit Assignment Models for Congested Networks," *Transportation Research Record* 2674, no. 1 (2020): 113–124, <https://doi.org/10.1177/0361198119900138>.
54. G. Yoon and J. Y. J. Chow, "A Sequential Transit Network Design Algorithm with Optimal Learning Under Correlated Beliefs," *Transportation Research Part E: Logistics and Transportation Review* 191 (2024): <https://doi.org/10.1016/j.tre.2024.103707>.
55. R. Oliveira Arbex and C. Barbieri da Cunha, "Efficient Transit Network Design and Frequencies Setting Multi-Objective Optimization by Alternating Objective Genetic Algorithm," *Transportation Research Part B: Methodological* 81 (2015): 355–376, <https://doi.org/10.1016/j.trb.2015.06.014>.
56. J. S. C. Chew, L. S. Lee, and H.-V. Seow, "Genetic Algorithm for Biobjective Urban Transit Routing Problem," *Journal of Applied Mathematics* 2013 (2013): 1–15, <https://doi.org/10.1155/2013/698645>.
57. M. Owais, M. K. Osman, and G. Moussa, "Multi-Objective Transit Route Network Design as Set Covering Problem," *IEEE Transactions on Intelligent Transportation Systems* 17, no. 3 (2016): 670–679, <https://doi.org/10.1109/TITS.2015.2480885>.
58. M. Owais and M. K. Osman, "Complete Hierarchical Multi-Objective Genetic Algorithm for Transit Network Design Problem," *Expert Systems with Applications* 114 (2018): 143–154, <https://doi.org/10.1016/j.eswa.2018.07.033>.
59. K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation* 6, no. 2 (2002): 182–197, <https://doi.org/10.1109/4235.996017>.
60. C. L. Mumford, "New Heuristic and Evolutionary Operators for the Multi-Objective Urban Transit Routing Problem," in *2013 IEEE Congress on Evolutionary Computation* (2013), 939–946, <https://doi.org/10.1109/CEC.2013.6557668>.
61. S. Chai and Q. Liang, "An Improved NSGA-II Algorithm for Transit Network Design and Frequency Setting Problem," *Journal of Advanced Transportation* 2020 (2020): 2020–20, <https://doi.org/10.1155/2020/2895320>.

62. P. N. Kechagiopoulos and G. N. Beligiannis, "Solving the Urban Transit Routing Problem Using a Particle Swarm Optimization Based Algorithm," *Applied Soft Computing* 21 (2014): 654–676, <https://doi.org/10.1016/j.asoc.2014.04.005>.
63. C. Iliopoulou and K. Kepaptsoglou, "Robust Electric Transit Route Network Design Problem (RE-TRNDP) With Delay Considerations: Model and Application," *Transportation Research Part C: Emerging Technologies* 129 (2021): 968–990X, <https://doi.org/10.1016/j.trc.2021.103255>.
64. Y. Liu, X. Feng, L. Zhang, W. Hua, and K. Li, "A Pareto Artificial Fish Swarm Algorithm for Solving a Multi-Objective Electric Transit Network Design Problem," *Transportmetrica A: Transportation Science* 16, no. 3 (2020): 1648–1670, <https://doi.org/10.1080/23249935.2020.1773574>.
65. K. Deb and H. Jain, "An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints," *IEEE Transactions on Evolutionary Computation* 18, no. 4 (2014): 577–601, <https://doi.org/10.1109/TEVC.2013.2281535>.
66. Q. Zhang and H. Li, "MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition," *IEEE Transactions on Evolutionary Computation* 11, no. 6 (2007): 712–731, <https://doi.org/10.1109/TEVC.2007.892759>.
67. W. Simon and B. Doerr, "A Mathematical Runtime Analysis of the Non-Dominated Sorting Genetic Algorithm III (NSGA-III)," in *GECCO'24 Companion: Proceedings of the Genetic and Evolutionary Computation Conference Companion* (2024), 63–64, <https://doi.org/10.1145/3638530.3664062>.
68. A. Opris, D.-C. Dang, F. Neumann, and D. Sudholt, "Runtime Analyses of NSGA-III on many-objective Problems," (2024), <https://arxiv.org/abs/2404.11433>.
69. M. Huang and K. Li, "A Survey of Decomposition-Based Evolutionary Multi-Objective Optimization: Part II—A Data Science Perspective," (2024), <https://arxiv.org/abs/2404.14228>.
70. B. Doerr, M. S. Krejca, and W. Noé, "Proven Runtime Guarantees for How the MOEA/D Computes the Pareto Front From the Subproblem Solutions," (2024), <https://arxiv.org/abs/2405.01014>.
71. M. Liang, M. Xu, and S. Wang, "A Novel Multi-Objective Evolutionary Algorithm for Transit Network Design and Frequency-Setting Problem Considering Passengers' Choice Behaviors Under Station Congestion," *Transportation Research Part B: Methodological* 197 (2025): <https://doi.org/10.1016/j.trb.2025.103238>.
72. I. C. Aksoy and Y. Alver, "Addressing Electric Transit Network Design Frequency Setting Problem With Dynamic Transit Assignment," *Transportation Business: Transport Dynamics* 12, no. 1 (2024): <https://doi.org/10.1080/21680566.2024.2318566>.
73. A. Holliday, A. El-Geneidy, and G. Dudek, "Learning Heuristics for Transit Network Design and Improvement With Deep Reinforcement Learning," (2025), <https://arxiv.org/abs/2404.05894>.
74. O. A. Nnene, J. W. Joubert, and M. H. Zuidgeest, "A Simulation-Based Optimization Approach for Designing Transit Networks," *Public Transport* 15, no. 2 (2023): 377–409, <https://doi.org/10.1007/s12469-022-00312-5>.
75. M. Owais, "Transit Network Design Problem: A Half Century of Methodological Research," *Innovative Infrastructure Solutions* 11, no. 1 (2025): 2364–4184, <https://doi.org/10.1007/s41062-025-02356-5>.
76. J. M. Ryan, *Urban Transportation Planning System (UTPS): the Community Aggregate Planning Model (CAPM) Users' Guide. Report-Urban Mass Transportation Administration* (Federal Highway Administration, Urban Planning Division, Technical Branch, 1979).
77. W. H. K. Lam, Z. C. Li, H. J. Huang, and S. C. Wong, "Modeling Time-Dependent Travel Choice Problems in Road Networks With Multiple User Classes and Multiple Parking Facilities," *Transportation Research Part B: Methodological* 40, no. 5 (2006): 368–395, <https://doi.org/10.1016/j.trb.2005.05.003>.
78. A. Tirachini, D. A. Hensher, and J. M. Rose, "Crowding in Public Transport Systems: Effects on Users, Operation and Implications for the Estimation of Demand," *Transportation Research Part A: Policy and Practice* 53 (2013): 36–52, <https://doi.org/10.1016/j.tra.2013.06.005>.
79. T. Brands, E. de Romph, T. Veitch, and J. Cook, "Modelling Public Transport Route Choice, With Multiple Access and Egress Modes," *Transportation Research Procedia* 1, no. 1 (2014): 12–23, <https://doi.org/10.1016/j.trpro.2014.07.003>.
80. M. Ben-Akiva and M. Bierlaire, *Discrete Choice Methods and Their Applications to Short Term Travel Decisions* (Springer, 1999), https://doi.org/10.1007/978-1-4615-5203-1_2.
81. Anita Schöbel and Reena Urban, "Fare Structure Design in Public Transport," (2025), <https://arxiv.org/abs/2502.08228>.
82. S. M. H. M. Moghaddam, K. Ramachandra Rao, G. Tiwari, and P. Biyani, "Simultaneous Bus Transit Route Network and Frequency Setting Search Algorithm," *Journal of Transportation Engineering* 145, no. 4 (2019): <https://doi.org/10.1061/JTEPBS.0000229>.
83. Y. Jin, "Finding the K Shortest Loopless Paths in a Network," *Management Science* 17, no. 11 (1971): 712–716, <https://doi.org/10.1287/mnsc.17.11.712>.
84. M. Nikolić and D. Teodorović, "Transit Network Design by Bee Colony Optimization," *Expert Systems With Applications* 40, no. 15 (2013): 5945–5955, <https://doi.org/10.1016/j.eswa.2013.05.002>.
85. A. Ceder, "Methods for Creating Bus Timetables," *Transportation Research Part A: General* 21, no. 1 (1987): 59–83, [https://doi.org/10.1016/0191-2607\(87\)90024-0](https://doi.org/10.1016/0191-2607(87)90024-0).
86. M. Nikolić and D. Teodorović, "A Simultaneous Transit Network Design and Frequency Setting: Computing With Bees," *Expert Systems With Applications* 41, no. 16 (2014): 7200–7209, <https://doi.org/10.1016/j.eswa.2014.05.034>.
87. M. A. Nayeem, M. Khaledur Rahman, and M. Sohel Rahman, "Transit Network Design by Genetic Algorithm With Elitism," *Transportation Research Part C: Emerging Technologies* 46 (2014): 30–45, <https://doi.org/10.1016/j.trc.2014.05.002>.
88. C. E. Mandl, "Evaluation and Optimization of Urban Public Transportation Networks," *European Journal of Operational Research* 5, no. 6 (1980): 396–404, [https://doi.org/10.1016/0377-2217\(80\)90126-5](https://doi.org/10.1016/0377-2217(80)90126-5).
89. X. Shen, S. Feng, Z. Li, and B. Hu, "Analysis of Bus Passenger Comfort Perception Based on Passenger Load Factor and in-Vehicle Time," *SpringerPlus* 5, no. 1 (2016): 62, <https://doi.org/10.1186/s40064-016-1694-7>.
90. M. Wardman, V. P. K. Chintakayala, and G. de Jong, "Values of Travel Time in Europe: Review and Meta-Analysis," *Transportation Research Part A: Policy and Practice* 94 (2016): 93–111, <https://doi.org/10.1016/j.tra.2016.08.019>.
91. M. Kouwenhoven, J. Muller, S. Thoen, J. Willigers, and G. de Jong, "Values of Time, Reliability and Comfort in the Netherlands 2022," *Technical Report, The Netherlands Institute for Transport Policy Analysis* (2023).
92. L. Maurer, F. Roca, N. Treffinger, L. Henke, V. Hofmann, and O. Leonhartsberger, "Tracking Parking Search and Occupancy in Zurich" (2023).
93. G. Mingardo, S. Vermeulen, and A. Bornioli, "Parking Pricing Strategies and Behaviour: Evidence From the Netherlands," *Transportation Research Part A: Policy and Practice* 157 (2022): 185–197, <https://doi.org/10.1016/j.tra.2022.01.005>.
94. Cargopedia, "European Fuel Prices," (2024), <https://www.cargopedia.net/europe-fuel-prices>.
95. T. Brands, *Multi-Objective Optimisation of Multimodal Passenger Transportation Networks* (Universiteit Twente, 2015), <https://research.utwente.nl/en/publications/multi-objective-optimisation-of-multimodal-passenger-transportati/>.

96. A. Tirachini, D. A. Hensher, and S. R. Jara-Díaz, "Comparing Operator and Users Costs of Light Rail, Heavy Rail and Bus Rapid Transit Over a Radial Public Transport Network," *Research in Transportation Economics* 29, no. 1 (2010): 231–242, <https://doi.org/10.1016/j.retrec.2010.07.029>.
97. A. T. Buba and L. S. Lee, "A Differential Evolution for Simultaneous Transit Network Design and Frequency Setting Problem," *Expert Systems With Applications* 106 (2018): 277–289, <https://doi.org/10.1016/j.eswa.2018.04.011>.
98. Y. Cao, B. J. Smucker, and T. J. Robinson, "On Using the Hypervolume Indicator to Compare Pareto Fronts: Applications to Multi-Criteria Optimal Experimental Design," *Journal of Statistical Planning and Inference* 160 (2015): 60–74, <https://doi.org/10.1016/j.jspi.2014.12.004>.
99. E. Zitzler and L. Thiele, "Multiobjective Optimization Using Evolutionary Algorithms—A Comparative Case Study," in *Parallel Problem Solving from Nature—PPSN V*, ed. A. E. Eiben, T. Bäck, M. Schoenauer, and H.-P. Schwefel (Springer Berlin Heidelberg, 1998), 292–301, <https://doi.org/10.1007/BFb0056872>.
100. A. Carlos, C. Coello, and M. R. Sierra, "A Study of the Parallelization of a Coevolutionary Multi-Objective Evolutionary Algorithm," in *MICAI 2004: Advances in Artificial Intelligence*, ed. R. Monroy, G. Arroyo-Figueroa, L. E. Sucar, and H. Sossa (Springer Berlin Heidelberg, 2004), 688–697, https://doi.org/10.1007/978-3-540-24694-7_71.
101. H. Ritchie, "Which Form of Transport Has the Smallest Carbon Footprint?" *Our World in Data* (2023), <https://ourworldindata.org/travel-carbon-footprint>.
102. ACEA, "Vehicles on European Roads," *Technical Report* (2024), <https://www.acea.auto/publication/report-vehicles-on-european-roads/>.
103. M. Wardman, "Public Transport Values of Time," *Transport Policy* 11, no. 4 (2004): 363–377, <https://doi.org/10.1016/j.tranpol.2004.05.001>.
104. SURF, "Snellius User Knowledge Base," (2024), <https://servicedesk.surf.nl/wiki/spaces/WIKI/pages/30660184/Snellius>.

Appendix A: A Zoomed in Pareto Fronts

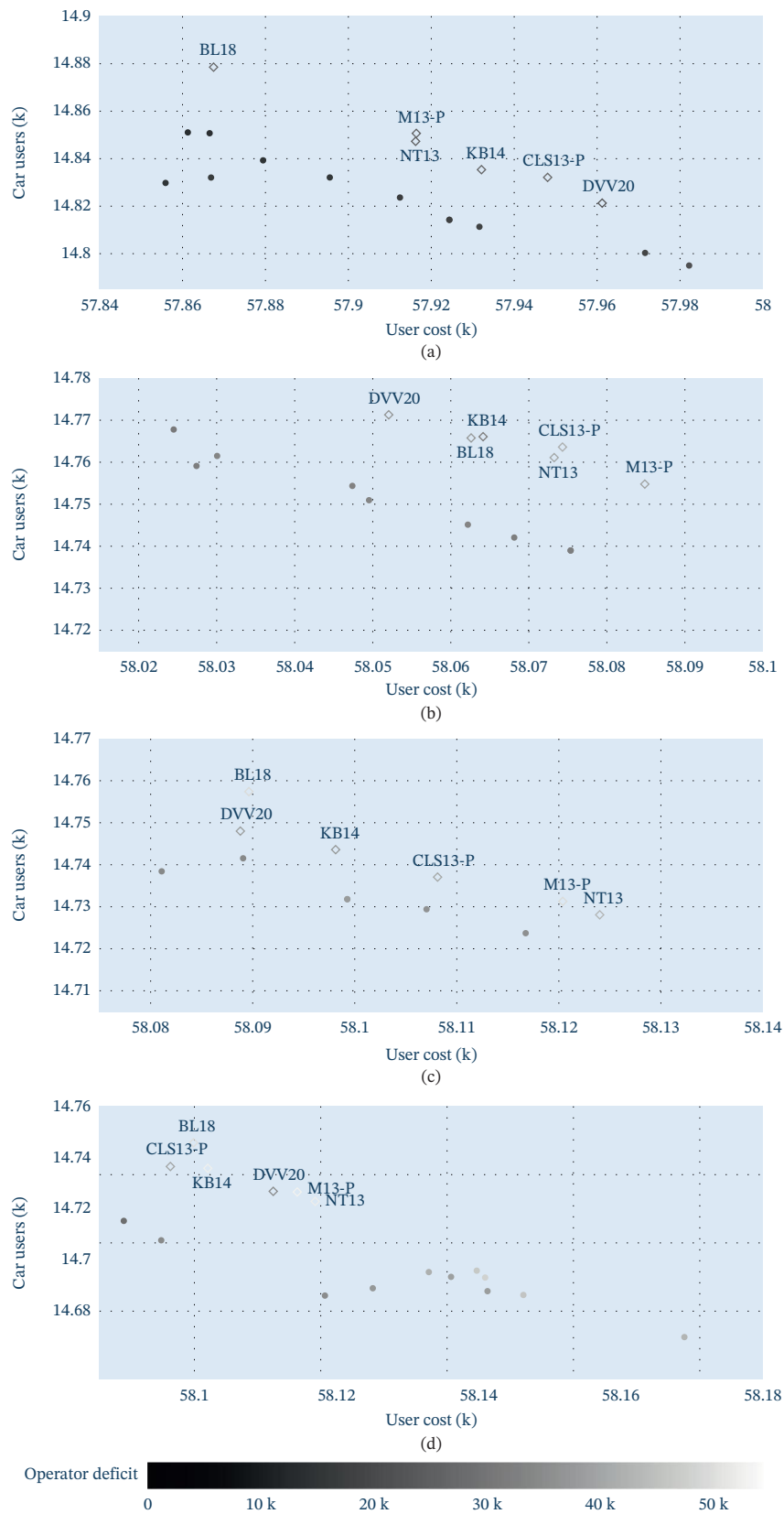


FIGURE A1 | Zoomed in parts of the Pareto fronts indicated by the dotted squares of Figure 5. (a) Zoomed Pareto front for four lines. (b) Zoomed Pareto front for six lines. (c) Zoomed Pareto front for seven lines. (d) Zoomed Pareto front for eight lines.

Appendix B: Solutions for Single-Objective Minimizers

TABLE A1 | Line configurations and parking fees for all solutions minimizing a single objective.

#lines	Solution	Line routes, line frequencies and parking fees
4	UC-MIN	l_1 : 3-6-8-15-7, $f^{l_1} = 10$ l_2 : 3-2-4-5, $f^{l_2} = 10$ l_3 : 14-10-11-13, $f^{l_3} = 10$ l_4 : 1-2-3-6-8, $f^{l_4} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
4	UC-MIN-F	l_1 : 10-11-13-14, $f^{l_1} = 2$ l_2 : 7-15-8-6-4-12-11, $f^{l_2} = 2$ l_3 : 5-4-2-3, $f^{l_3} = 2$ l_4 : 11-13-14, $f^{l_4} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
4	UC-MIN-FP	l_1 : 10-11-13-14, $f^{l_1} = 2$ l_2 : 6-8-15-7-10, $f^{l_2} = 2$ l_3 : 1-2-3-6-8, $f^{l_3} = 2$ l_4 : 5-4-6, $f^{l_4} = 2$ $p^{1,\dots,15} = [0.25, \dots, 0.25]$
4	CU-MIN	l_1 : 1-2-4-6-8-10-14-13, $f^{l_1} = 10$ l_2 : 5-4-6-15-7-10-11-12, $f^{l_2} = 10$ l_3 : 1-2-3-6-8-10-11-13, $f^{l_3} = 10$ l_4 : 1-2-3-6-8-10-11-13, $f^{l_4} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
4	CU-MIN-F	l_1 : 8-6-15-7-10-11-13-14, $f^{l_1} = 20$ l_2 : 10-7-15-8-6-4-12-11, $f^{l_2} = 20$ l_3 : 5-4-2-3-6-8-10-7, $f^{l_3} = 20$ l_4 : 1-2-3-6-8-10-14-13, $f^{l_4} = 20$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
4	CU-MIN-FP	l_1 : 2-3-6-8-10-11-13-14, $f^{l_1} = 20$ l_2 : 1-2-5-4-6-8-10-7, $f^{l_2} = 20$ l_3 : 1-2-4-12-11-10-13-14, $f^{l_3} = 20$ l_4 : 2-3-6-8-15-7-10-11, $f^{l_4} = 20$ $p^{1,\dots,15} = [2.5, \dots, 2.5]$
4	OD-MIN	l_1 : 3-2-4-5, $f^{l_1} = 10$ l_2 : 3-6-8-15-7-10, $f^{l_2} = 10$ l_3 : 10-11-13-14, $f^{l_3} = 10$ l_4 : 1-2-3, $f^{l_4} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
4	OD-MIN-F	l_1 : 5-4-2-3, $f^{l_1} = 2$ l_2 : 3-6-8-15-7, $f^{l_2} = 2$ l_3 : 3-6-15-7-10, $f^{l_3} = 2$ l_4 : 10-11-13-14, $f^{l_4} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
4	OD-MIN-FP	l_1 : 1-2-3-6-8, $f^{l_1} = 4$ l_2 : 6-15-7-10, $f^{l_2} = 4$ l_3 : 10-11-13-14, $f^{l_3} = 12$ l_4 : 5-4-6, $f^{l_4} = 10$ $p^{1,\dots,15} = [2.25, 2.5, 2.5, 2.5, 2.5, 2.5, 0.25, 2.5, 0.75, 2.5, 2.5, 0.25, 2.5, 0.25, 1.5]$
6	UC-MIN	l_1 : 8-15-7-10, $f^{l_1} = 10$ l_2 : 5-4-2-3-6-8, $f^{l_2} = 10$ l_3 : 11-13-14, $f^{l_3} = 10$ l_4 : 1-2, $f^{l_4} = 10$ l_5 : 2-4-12, $f^{l_5} = 10$ l_6 : 6-15-7-10, $f^{l_6} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$

(Continues)

TABLE A1 | (Continued)

#lines	Solution	Line routes, line frequencies and parking fees
6	UC-MIN-F	l_1 : 3-6-4-12, $f^{l_1} = 2$ l_2 : 12-11-13-14, $f^{l_2} = 2$ l_3 : 6-8-15-9, $f^{l_3} = 2$ l_4 : 1-2-4, $f^{l_4} = 2$ l_5 : 7-10-11, $f^{l_5} = 2$ l_6 : 5-4-12, $f^{l_6} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
6	UC-MIN-FP	l_1 : 9-15-7-10, $f^{l_1} = 2$ l_2 : 4-6-8-10, $f^{l_2} = 2$ l_3 : 10-11-13-14, $f^{l_3} = 2$ l_4 : 5-4-2-3, $f^{l_4} = 2$ l_5 : 10-11-12, $f^{l_5} = 2$ l_6 : 1-2-3-6-15-7, $f^{l_6} = 2$ $p^{1,\dots,15} = [0.25, \dots, 0.25]$
6	CU-MIN	l_1 : 1-2-4-12-11-10-13-14, $f^{l_1} = 10$ l_2 : 1-2-3-6-8-10-14-13, $f^{l_2} = 10$ l_3 : 8-6-15-7-10-11-13-14, $f^{l_3} = 10$ l_4 : 5-4-6-15-7-10-11-13, $f^{l_4} = 10$ l_5 : 1-2-3-6-8-10-11-13, $f^{l_5} = 10$ l_6 : 5-4-2-3-6-8-10-11, $f^{l_6} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
6	CU-MIN-F	l_1 : 1-2-3-6-8-15-9, $f^{l_1} = 20$ l_2 : 5-4-2-3-6-8-10-7, $f^{l_2} = 20$ l_3 : 1-2-4-6-8-10-11-13, $f^{l_3} = 20$ l_4 : 8-6-15-7-10-11-13-14, $f^{l_4} = 20$ l_5 : 1-2-4-12-11-10-13-14, $f^{l_5} = 20$ l_6 : 1-2-3-6-8-10-11-13, $f^{l_6} = 20$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
6	CU-MIN-FP	l_1 : 5-4-2-3-6-8-10-7, $f^{l_1} = 20$ l_2 : 9-15-7-10-11-12-4-6, $f^{l_2} = 20$ l_3 : 8-6-15-7-10-11-13-14, $f^{l_3} = 20$ l_4 : 1-2-3-6-15-7-10-8, $f^{l_4} = 20$ l_5 : 2-5-4-6-8-10-11-13, $f^{l_5} = 20$ l_6 : 1-2-4-6-8-10-11-13, $f^{l_6} = 20$ $p^{1,\dots,15} = [2.5, \dots, 2.5]$
6	OD-MIN	l_1 : 5-4-2-3-6-8, $f^{l_1} = 10$ l_2 : 6-15-7, $f^{l_2} = 10$ l_3 : 1-2, $f^{l_3} = 10$ l_4 : 4-12, $f^{l_4} = 10$ l_5 : 8-15-7-10, $f^{l_5} = 10$ l_6 : 10-11-13-14, $f^{l_6} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
6	OD-MIN-F	l_1 : 8-6-3-2-4, $f^{l_1} = 2$ l_2 : 1-2-5, $f^{l_2} = 2$ l_3 : 6-8-15-9, $f^{l_3} = 2$ l_4 : 2-4-12, $f^{l_4} = 2$ l_5 : 15-7-10-11, $f^{l_5} = 2$ l_6 : 11-13-14, $f^{l_6} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
6	OD-MIN-FP	l_1 : 10-11-12, $f^{l_1} = 2$ l_2 : 2-3-6-8-10, $f^{l_2} = 9$ l_3 : 9-15-7-10, $f^{l_3} = 4$ l_4 : 1-2-4, $f^{l_4} = 4$ l_5 : 10-11-13-14, $f^{l_5} = 4$ l_6 : 5-4, $f^{l_6} = 4$ $p^{1,\dots,15} = [0.25, 2.25, 1.5, 0.25, 0.25, 2.5, 2.5, 2.5, 1.0, 2.5, 2.5, 0.25, 2.5, 1.75, 1.0]$

(Continues)

TABLE A1 | (Continued)

#lines	Solution	Line routes, line frequencies and parking fees
7	UC-MIN	l_1 : 8-6-3-2-4, $f^{l_1} = 10$ l_2 : 5-4-12, $f^{l_2} = 10$ l_3 : 1-2-4, $f^{l_3} = 10$ l_4 : 8-6-15-9, $f^{l_4} = 10$ l_5 : 9-15-7, $f^{l_5} = 10$ l_6 : 10-11-13-14, $f^{l_6} = 10$ l_7 : 10-11-13-14, $f^{l_7} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
7	UC-MIN-F	l_1 : 4-6-8-15-9, $f^{l_1} = 2$ l_2 : 15-7-10-11, $f^{l_2} = 2$ l_3 : 5-4-2-3, $f^{l_3} = 2$ l_4 : 11-13-14, $f^{l_4} = 2$ l_5 : 1-2, $f^{l_5} = 2$ l_6 : 4-12, $f^{l_6} = 2$ l_7 : 4-12, $f^{l_7} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
7	UC-MIN-FP	l_1 : 3-2-4-5, $f^{l_1} = 2$ l_2 : 5-4-6-15-7, $f^{l_2} = 2$ l_3 : 7-10-11-13-14, $f^{l_3} = 2$ l_4 : 1-2, $f^{l_4} = 2$ l_5 : 8-15-9, $f^{l_5} = 2$ l_6 : 9-15-7-10-11-12, $f^{l_6} = 2$ l_7 : 7-15-6-3-2-4-12, $f^{l_7} = 2$ $p^{1,\dots,15} = [0.25, \dots, 0.25]$
7	CU-MIN	l_1 : 1-2-3-6-8-10-11-13, $f^{l_1} = 10$ l_2 : 5-4-2-3-6-15-7-10, $f^{l_2} = 10$ l_3 : 1-2-4-12-11-10-13-14, $f^{l_3} = 10$ l_4 : 9-15-7-10-11-13-14, $f^{l_4} = 10$ l_5 : 1-2-3-6-8-10-14-13, $f^{l_5} = 10$ l_6 : 5-4-2-3-6-8-10-7, $f^{l_6} = 10$ l_7 : 1-2-4-6-8-15-7-10, $f^{l_7} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
7	CU-MIN-F	l_1 : 1-2-3-6-8-10-11-13, $f^{l_1} = 20$ l_2 : 9-15-7-10-11-13-14, $f^{l_2} = 20$ l_3 : 1-2-4-5, $f^{l_3} = 20$ l_4 : 7-10-14-13-11-12-4-6, $f^{l_4} = 20$ l_5 : 11-10-8-6-3-2-4-12, $f^{l_5} = 20$ l_6 : 1-2-3-6-8-10-11-13, $f^{l_6} = 20$ l_7 : 8-6-15-7-10-11-13-14, $f^{l_7} = 20$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
7	CU-MIN-FP	l_1 : 10-7-15-6-3-2-4-12, $f^{l_1} = 20$ l_2 : 2-4-6-8-10-14-13-11, $f^{l_2} = 20$ l_3 : 1-2-5-4-6-3, $f^{l_3} = 20$ l_4 : 15-7-10-11-12-4-6-8, $f^{l_4} = 20$ l_5 : 14-13-11-10-8-15-9, $f^{l_5} = 20$ l_6 : 2-5-4-6-8-10-11-13, $f^{l_6} = 20$ l_7 : 1-2-3-6-8-10-14-13, $f^{l_7} = 20$ $p^{1,\dots,15} = [2.5, \dots, 2.5]$
7	OD-MIN	l_1 : 1-2-4, $f^{l_1} = 10$ l_2 : 5-4-12, $f^{l_2} = 10$ l_3 : 4-2-3-6-8, $f^{l_3} = 10$ l_4 : 8-15-9, $f^{l_4} = 10$ l_5 : 15-7-10-11, $f^{l_5} = 10$ l_6 : 11-13-14, $f^{l_6} = 10$ l_7 : 11-13-14, $f^{l_7} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$

(Continues)

TABLE A1 | (Continued)

#lines	Solution	Line routes, line frequencies and parking fees
7	OD-MIN-F	l_1 : 4-12, $f^{l_1} = 2$ l_2 : 5-4-2-3-6, $f^{l_2} = 2$ l_3 : 8-6-15-9, $f^{l_3} = 2$ l_4 : 6-8-10-11, $f^{l_4} = 2$ l_5 : 11-13-14, $f^{l_5} = 2$ l_6 : 1-2, $f^{l_6} = 2$ l_7 : 1-2, $f^{l_7} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
7	OD-MIN-FP	l_1 : 10-11-13-14, $f^{l_1} = 2$ l_2 : 2-3-6-8-10, $f^{l_2} = 7$ l_3 : 9-15-7-10, $f^{l_3} = 2$ l_4 : 1-2-4, $f^{l_4} = 7$ l_5 : 11-12, $f^{l_5} = 5$ l_6 : 2-4-5, $f^{l_6} = 7$ l_7 : 2-4-5, $f^{l_7} = 5$ $p^{1,\dots,15} = [2.5, 2.5, 2.5, 0.5, 2.0, 2.5, 1.5, 0.75, 0.25, 2.5, 0.75, 0.5, 0.25, 2.0, 1.75]$
8	UC-MIN	l_1 : 10-11-13-14, $f^{l_1} = 10$ l_2 : 4-2-3-6-8, $f^{l_2} = 10$ l_3 : 9-15-7-10, $f^{l_3} = 10$ l_4 : 1-2, $f^{l_4} = 10$ l_5 : 3-2-4-12, $f^{l_5} = 10$ l_6 : 5-4-2-3-6-8, $f^{l_6} = 10$ l_7 : 1-2, $f^{l_7} = 10$ l_8 : 9-15-7-10, $f^{l_8} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
8	UC-MIN-F	l_1 : 8-15-6-4-12, $f^{l_1} = 2$ l_2 : 9-15-7-10, $f^{l_2} = 2$ l_3 : 10-11-13-14, $f^{l_3} = 2$ l_4 : 1-2-4-12, $f^{l_4} = 2$ l_5 : 2-5-4-6, $f^{l_5} = 2$ l_6 : 2-3-6-8, $f^{l_6} = 2$ l_7 : 9-15-7-10, $f^{l_7} = 2$ l_8 : 5-4-12, $f^{l_8} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
8	UC-MIN-FP	l_1 : 2-4-12, $f^{l_1} = 2$ l_2 : 5-4-6-8-10, $f^{l_2} = 2$ l_3 : 9-15-7-10, $f^{l_3} = 2$ l_4 : 10-11-13-14, $f^{l_4} = 2$ l_5 : 1-2, $f^{l_5} = 2$ l_6 : 5-4-2-3-6-8, $f^{l_6} = 2$ l_7 : 12-11-13, $f^{l_7} = 2$ l_8 : 1-2-4, $f^{l_8} = 2$ $p^{1,\dots,15} = [0.25, \dots, 0.25]$
8	CU-MIN	l_1 : 5-4-6-8-10-14-13-11, $f^{l_1} = 10$ l_2 : 1-2-3-6-15-7-10-11, $f^{l_2} = 10$ l_3 : 1-2-4-12-11-10-13-14, $f^{l_3} = 10$ l_4 : 9-15-8-6-3-2-4-12, $f^{l_4} = 10$ l_5 : 1-2-3-6-15-7-10-11, $f^{l_5} = 10$ l_6 : 5-4-6-8-10-14-13-11, $f^{l_6} = 10$ l_7 : 2-3-6-8-10-11-13-14, $f^{l_7} = 10$ l_8 : 1-2-3-6-15-7-10-14, $f^{l_8} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$

(Continues)

TABLE A1 | (Continued)

#lines	Solution	Line routes, line frequencies and parking fees
8	CU-MIN-F	l_1 : 5-4-6-8-10-14-13-11, $f^{l_1} = 20$ l_2 : 1-2-4-12-11-10-13-14, $f^{l_2} = 20$ l_3 : 9-15-7-10-11-13-14, $f^{l_3} = 20$ l_4 : 1-2-3-6-8-10-11-13, $f^{l_4} = 20$ l_5 : 8-6-15-7-10-11-13-14, $f^{l_5} = 20$ l_6 : 5-4-2-3-6-8-10-7, $f^{l_6} = 20$ l_7 : 3-6-8-15-7-10-11-13, $f^{l_7} = 20$ l_8 : 5-4-6-15-7-10-11-13, $f^{l_8} = 20$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
8	CU-MIN-FP	l_1 : 9-15-7-10-11-13-14, $f^{l_1} = 20$ l_2 : 1-2-3-6-8-10-14-13, $f^{l_2} = 20$ l_3 : 1-2-4-12-11-10-13-14, $f^{l_3} = 20$ l_4 : 14-13-10-11-12-4-6-8, $f^{l_4} = 20$ l_5 : 2-5-4-6-8-10-11-13, $f^{l_5} = 20$ l_6 : 9-15-6-3-2-4-12-11, $f^{l_6} = 20$ l_7 : 1-2-3-6-8-10-11-13, $f^{l_7} = 20$ l_8 : 4-6-8-15-7-10-11-13, $f^{l_8} = 20$ $p^{1,\dots,15} = [2.5, \dots, 2.5]$
8	OD-MIN	l_1 : 4-5, $f^{l_1} = 10$ l_2 : 4-2-3-6-8-15-7, $f^{l_2} = 10$ l_3 : 7-10-11, $f^{l_3} = 10$ l_4 : 11-13-14, $f^{l_4} = 10$ l_5 : 1-2, $f^{l_5} = 10$ l_6 : 4-12, $f^{l_6} = 10$ l_7 : 6-15-9, $f^{l_7} = 10$ l_8 : 4-5, $f^{l_8} = 10$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
8	OD-MIN-F	l_1 : 1-2-4, $f^{l_1} = 2$ l_2 : 5-4-6, $f^{l_2} = 2$ l_3 : 2-3-6-15-7-10, $f^{l_3} = 2$ l_4 : 10-11-13-14, $f^{l_4} = 2$ l_5 : 11-12, $f^{l_5} = 2$ l_6 : 9-15-7, $f^{l_6} = 2$ l_7 : 2-3-6-8, $f^{l_7} = 2$ l_8 : 3-6-8-15-7-10, $f^{l_8} = 2$ $p^{1,\dots,15} = [1.25, \dots, 1.25]$
8	OD-MIN-FP	l_1 : 2-3-6-8-10, $f^{l_1} = 7$ l_2 : 9-15-7-10, $f^{l_2} = 2$ l_3 : 10-11-13-14, $f^{l_3} = 7$ l_4 : 5-4-6, $f^{l_4} = 7$ l_5 : 11-12, $f^{l_5} = 5$ l_6 : 1-2-4, $f^{l_6} = 7$ l_7 : 1-2-3-6, $f^{l_7} = 2$ l_8 : 2-3-6-8-10-11-13, $f^{l_8} = 2$ $p^{1,\dots,15} = [0.5, 2.5, 2.5, 1.75, 1.0, 2.5, 0.25, 2.5, 1.0, 2.5, 2.5, 0.25, 2.5, 0.5, 0.75]$

Note: In the constant-frequencies and constant-parking cases, $f^{l_i} = 10$ for all lines and $p^{1,\dots,15} = [1.25, \dots, 1.25]$ (fixed, not decision variables). For constant parking, $p^{1,\dots,15} = [1.25, \dots, 1.25]$.

Appendix C: Supporting Tables Sensitivity Analysis

TABLE A2 | Results for $c_a = 1500$.

Solution	User costs	Car users	Operator deficit
UC-MIN	69,167	14,308	11,546
UC-MIN-F	68,867	14,433	8322
UC-MIN-FP	54,408	15,469	1017
CU-MIN	69,304	14,257	15,159
CU-MIN-F	69,159	14,136	35,080
CU-MIN-FP	85,844	11,985	36,862
OD-MIN	69,526	14,476	3210
OD-MIN-F	70,069	15,444	327
OD-MIN-FP	81,053	13,212	-1804

TABLE A3 | Results for different values of $\mu_{a,e}$.

$\mu_{a,e}$	Solution	User costs	Car users	Operator deficit
1	UC-MIN	56,498	15,405	2942
	UC-MIN-F	56,358	15,432	408
	UC-MIN-FP	40,633	15,495	390
	CU-MIN	58,987	13,750	15,890
	CU-MIN-F	59,034	13,589	30,809
	CU-MIN-FP	77,559	10,480	34,585
	OD-MIN	57,987	14,445	243
	OD-MIN-F	57,650	14,765	-91
	OD-MIN-FP	70,596	12,775	-2562
3	UC-MIN	56,498	15,405	2942
	UC-MIN-F	55,959	15,559	893
	UC-MIN-FP	40,374	15,562	405
	CU-MIN	56,942	15,282	16,116
	CU-MIN-F	57,082	15,237	41,362
	CU-MIN-FP	77,868	14,499	39,139
	OD-MIN	56,612	15,386	2801
	OD-MIN-F	56,092	15,530	556
	OD-MIN-FP	40,374	15,562	405

TABLE A4 | Results for $\text{VoT}_{\text{search}} = \text{VoT}_{\text{walk}} = \text{VoT}_{\text{cycle}} = \text{VoT}_{\text{wait}} = 15$.

Solution	User costs	Car users	Operator deficit
UC-MIN	56,744	15,263	3351
UC-MIN-F	55,966	15,558	1701
UC-MIN-FP	40,372	15,562	439
CU-MIN	57,602	14,982	17,117
CU-MIN-F	57,776	14,883	31,894
CU-MIN-FP	78,666	13,470	35,377
OD-MIN	56,837	15,263	2492
OD-MIN-F	55,970	15,557	399
OD-MIN-FP	40,372	15,562	439

TABLE A5 | Results for different values of T_{park} .

T_{park}	Solution	User costs	Car users	Operator deficit
1	UC-MIN	46,955	15,343	1994
	UC-MIN-F	46,259	15,552	964
	UC-MIN-FP	38,517	15,546	628
	CU-MIN	47,474	15,172	12,102
	CU-MIN-F	47,691	15,095	45,249
	CU-MIN-FP	58,119	14,707	35,458
	OD-MIN	46,955	15,343	1994
	OD-MIN-F	46,335	15,538	469
	OD-MIN-FP	38,517	15,546	628
3	UC-MIN	67,323	14,765	3050
	UC-MIN-F	66,101	15,450	215
	UC-MIN-FP	42,484	15,531	1751
	CU-MIN	68,354	14,271	14,719
	CU-MIN-F	68,548	14,087	34,089
	CU-MIN-FP	96,076	9633	33,703
	OD-MIN	67,371	14,802	630
	OD-MIN-F	66,449	15,303	135
	OD-MIN-FP	85,860	11,970	-2130

TABLE A6 | Results for different values of O .

O	Solution	User costs	Car users	Operator deficit
10	UC-MIN	56,916	15,291	6567
	UC-MIN-F	56,155	15,516	157
	UC-MIN-FP	40,518	15,533	440
	CU-MIN	57,882	14,826	7334
	CU-MIN-F	58,118	14,709	18,833
	CU-MIN-FP	78,696	13,072	15,860
	OD-MIN	57,281	15,099	518
	OD-MIN-F	56,296	15,470	97
	OD-MIN-FP	72,448	14,513	-886
30	UC-MIN	56,879	15,301	24,273
	UC-MIN-F	56,137	15,519	780
	UC-MIN-FP	40,469	15,544	827
	CU-MIN	57,902	14,822	29,917
	CU-MIN-F	58,101	14,712	53,655
	CU-MIN-FP	78,823	13,032	55,470
	OD-MIN	57,145	15,126	3436
	OD-MIN-F	56,303	15,466	723
	OD-MIN-FP	65,142	15,208	675